MAT 136: Calculus I
Weekly Homework 2

NAME:

Instructions

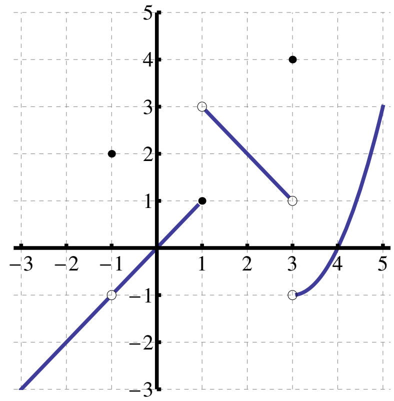
Complete each of the following exercises. Your solutions should be complete and neatly written. In particular, you should show all of your work. Write your solutions on your own paper or prepare them digitally. You will need to capture your work digitally and then upload a single PDF document (possibly with multiple pages) to BbLearn. There are many free smartphone apps for doing this. I use TurboScan on my iPhone. This assignment is due on **Thursday, August 27 by 8:00pm**.

Problems

1. When running a marathon you heard the timer call out 12 minutes as you passed the second mile-marker.
   (a) As you passed mile-marker 5 you heard the timer call out 33 minutes. What was your average speed from mile 2 to mile 5?
   (b) If you passed mile marker 5 at 33 minutes, what average speed do you need to run for the remainder of the race to meet your goal of completing the 26.2-mile marathon in 175 minutes? (Round your answer to two decimal places.)
2. Consider the function $f(x) = \frac{\sin(x)}{x}$.
   (a) Use a graph to make an educated guess at the following limit:
   $$\lim_{x\to 0} \frac{\sin(x)}{x}$$
   (b) Where is f continuous? Briefly justify your answer. You may rely on the graph.
3. Provide an example of a function for each of the following. You can either draw a graph or provide a formula for each example. If it is impossible to find an example, explain why.
   (a) $f(1)$ exists but $\lim_{x\to 1} f(x)$ does not exist.
   (b) $\lim_{x\to 1} f(x)$ exists but $f(1)$ does not exist.
   (c) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist but f is not continuous at $x = 1$.
   (d) f is continuous at $x = 1$ but one of $\lim_{x\to 1} f(x)$ or $f(1)$ does not exist.
4. Provide examples of functions f and g such that $\lim_{x\to 0}(f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.
5. Provide examples of functions f and g such that $\lim_{x\to 0}(f(x)g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.
6. Suppose f is a function such that f has a tangent line at $x = a$. Provide an intuitive explanation (i.e., not a formal proof) as to why this implies that f is continuous at $x = a$.*

*In Problem 4 on Weekly Homework 1, you provided an example of a function that was continuous at $x = 0$ but did not have a tangent line at $x = 0$. This shows that the converse of the stated claim is not true in general. That is, if a function f is continuous at $x = a$, the graph of f may or may not have a tangent line at $x = a$.

7. Let f be given by the following graph.



   (a) Identify any x-values where f has discontinuities.

   (b) Are there any values for which f is discontinuous but f is either left or right continuous? Explain your answer.

8. Find the values of the constants a and b that make the following function continuous.

$$f(x) = \begin{cases} \frac{1}{x}, & \text{for } x < -1 \\ ax + b, & \text{for } -1 \leq x \leq \frac{1}{2} \\ \frac{1}{x}, & \text{for } x > \frac{1}{2} \end{cases}$$

9. Evaluate each of the following limits. If a limit does not exist, specify whether it equals ∞, $-\infty$, or simply does not exist (in which case, write DNE). Sufficient work must be shown and proper notation should be used. In particular, you should write "lim" and "=" where appropriate. Give *exact answers*.

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x-6}+\frac{1}{6}}{x}$

   (c) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

   (d) $\displaystyle\lim_{x\to 2^+} \frac{-1}{x-2}$

   (e) $\displaystyle\lim_{x\to 2^-} \frac{|x-2|}{x-2}$