MAT 136: Calculus I
Weekly Homework 6

NAME:

Instructions

Complete each of the following exercises. Your solutions should be complete and neatly written. In particular, you should show all of your work. Write your solutions on your own paper or prepare them digitally. You will need to capture your work digitally and then upload a single PDF document (possibly with multiple pages) to BbLearn. There are many free smartphone apps for doing this. I use TurboScan on my iPhone. This assignment is due on **Friday, October 23 by 8:00pm**.

Problems

1. Let $f(x) = \dfrac{x}{x+2}$. It turns out that f satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 4]$ (you do not need to show this). Find all the numbers c that the Mean Value Theorem guarantees exist.

2. A truck driver handed in a ticket at a toll booth showing that in 2 hours they had covered 158 miles on a toll road with speed limit 70 mph. The driver was cited for speeding. Use the Mean Value Theorem to explain why. Be sure to state the assumptions that we have to make about the position function $p(t)$ of the truck to be able to apply the Mean Value Theorem.

3. Provide an example of a function f such that $f'(2) = 0$ but f does not have a local maximum or local minimum at $x = 2$. You can either draw a graph or provide a formula for your function.

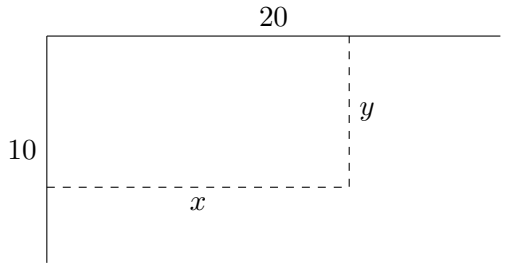
4. Provide an example of a function f such that $f''(1) = 0$ but f does not have an inflection point at $x = 1$. You can either draw a graph or provide a formula for your function.

5. Consider the function $f(x) = \dfrac{2x^2}{x^2 - 1}$. Answer each of the following questions using calculus.

   (a) Does f have any vertical asymptotes? If so, find them. If not, explain why.
   (b) Find the critical numbers of f.
   (c) Determine the intervals where f is increasing and the intervals where f is decreasing.
   (d) Classify each of the critical numbers of f as a local maximum, local minimum, or neither.
   (e) Find the values where $f''(x)$ equals 0 or does not exist.
   (f) Determine the intervals where f is concave up and the intervals where f is concave down.
   (g) Does f have any inflection points? If so, identify the corresponding x-values. If not, explain why.
   (h) Does f have any horizontal asymptotes? If so, find them. If not, explain why.
   (i) Using the information above, sketch the graph of f.

6. Use calculus to find the global maximum and global minimum of $f(x) = 2x + \dfrac{1}{2x}$ on the interval $[1, 4]$.

7. Suppose you want to build an enclosed, rectangular pen for your cute baby nugget. For two sides of the pen you are going to use two perpendicular stone walls in your backyard, whose total lengths are 10 ft and 20 ft, respectively, and for the other two sides you are going to use 24 ft of fencing. See figure.



Find the *dimensions* that will maximize the area of the rectangular pen. Justifying your answer will not only make sure that you receive full credit, but will also ensure that you don't make a mistake.

