Derivative Practice

1. $f(x) = \pi^2$, $f'(x) = 0$

2. $f(t) = 3e^{4t}$, $f'(t) = 12e^{4t}$

3. $g(w) = \dfrac{e^3}{3^w}$

$$g'(w) = \frac{3^w(0) - e^3(3^w \ln 3)}{(3^w)^2} = \frac{-e^3 3^w \ln 3}{3^{2w}} = \frac{-e^3 \ln 3}{3^w}$$

or re-write + use product rule:

$g(w) = e^3 3^{-w}$

$$g'(w) = e^3(-3^{-w}\ln 3) + 3^{-w}(0) = -\frac{e^3 \ln 3}{3^w}$$

4. $h(s) = e^{2s}\ln(2s)$

$$h'(s) = e^{2s}\left(\frac{1}{2s}\cdot 2\right) + \ln(2s)\left(2e^{2s}\right)$$

$$= \frac{e^{2s}}{s} + 2e^{2s}\ln(2s)$$

5. $f(x) = 5\sqrt{\log_3 x}$

$$f'(x) = 5\left(\frac{1}{2}\left(\log_3 x\right)^{-\frac{1}{2}}\left(\frac{1}{x\ln 3}\right)\right)$$

$$= \frac{5}{2x\ln 3\sqrt{\log_3 x}}$$

6. $g(x) = x^2 e^{x^2}$

$g'(x) = x^2\left(e^{x^2}(2x)\right) + e^{x^2}(2x)$

$= 2x^3 e^{x^2} + 2x\,e^{x^2} = 2xe^{x^2}(x^2+1)$

7. $f(x) = x^e$

$f'(x) = e x^{e-1}$

8. $f(x) = (\pi e)^2$ all constants!

$f'(x) = 0$

9. $m(t) = \tan(3t)$

$m'(t) = \left(\sec^2(3t)\right)(3) = 3\sec^2(3t)$

10. $g(y) = y\cos(\ln y)$

$g'(y) = y\left(-\sin(\ln y)\left(\frac{1}{y}\right)\right) + \cos(\ln y)$

$= -\sin(\ln y) + \cos(\ln y)$

11. $h(t) = t\sin t$

$h'(t) = t\cos t + \sin t$

12. $f(x) = \dfrac{x}{\sin x}$

$f'(x) = \dfrac{\sin x(1) - x\cos x}{\sin^2 x} = \dfrac{\sin x - \cos x}{\sin^2 x}$

13. $f(x) = \ln\left(\frac{x^{3/5}\sqrt{3-x}}{(x^2-4)^4}\right)$

$= \frac{3}{5}\ln x + \frac{1}{2}\ln(3-x) - 4\ln(x^2-4)$

$f'(x) = \frac{3}{5x} + \frac{1}{2}\left(\frac{1}{3-x}\right)(-1) - 4\left(\frac{1}{x^2-4}\right)(2x)$

$= \frac{3}{5x} - \frac{1}{2(3-x)} - \frac{8x}{x^2-4}$

14. $y = x^{\cos x} = e^{\ln x^{\cos x}} = e^{\cos x \ln x}$

$\frac{dy}{dx} = e^{\cos x \ln x}\left(\cos x\left(\frac{1}{x}\right) + (\ln x)(-\sin x)\right)$

$= e^{\cos x \ln x}\left(\frac{\cos x}{x} - (\ln x)(\sin x)\right)$

$= x^{\cos x}\left(\frac{\cos x}{x} - (\ln x)(\sin x)\right)$

15. $f(x) = e^{x^2}\cos(2x)\sqrt{3x+1}$

$f'(x) = e^{x^2}\left(\cos(2x)\left(\frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)\right) + \sqrt{3x+1}\left(-\sin(2x)(2)\right)\right)$

$+ \cos(2x)\sqrt{3x+1}\left(e^{x^2}(2x)\right)$

$= e^{x^2}\left[\frac{3\cos(2x)}{2\sqrt{3x+1}} - 2\sqrt{3x+1}\sin(2x)\right] + 2xe^{x^2}\cos(2x)\sqrt{3x+1}$

Derivatives in multiple ways

16) $f(x) = (x+1)(x^2-3)$

1) product rule:

$$f'(x) = (x+1)(2x) + (x^2-3)(1)$$

2) simplification:

$$f(x) = x^3 - 3x + x^2 - 3$$

$$f'(x) = 3x^2 - 3 + 2x$$

17) $g(x) = \dfrac{3x^2 + 5x}{\sqrt{x}}$

1) quotient rule:

$$g'(x) = \frac{\sqrt{x}\,(6x+5) + (3x^2+5x)(\frac{1}{2}x^{-1/2})}{(\sqrt{x})^2}$$

2) Simplification:

$$g(x) = \frac{3x^2}{x^{1/2}} + \frac{5x}{x^{1/2}}$$

$$= 3x^{3/2} + 5x^{1/2}$$

$$g'(x) = \frac{9}{2}x^{1/2} + \frac{5}{2}x^{-1/2}$$

18) $y = \dfrac{x^2-1}{x}$

1) Quotient rule:

$$y' = \frac{(x)(2x) - (x^2-1)(1)}{x^2}$$

2) simplification

$$y = \frac{x^2}{x} - \frac{1}{x}$$

$$= x - x^{-1}$$

$$y' = 1 + x^{-2}$$

19) $f(x) = \dfrac{7x + 3x^2}{5\sqrt{x}}$

1) quotient rule:

$$f'(x) = \frac{5\sqrt{x}\,(7 + 6x) - (7x + 3x^2)\left(\frac{5}{2}x^{-1/2}\right)}{(5\sqrt{x})^2}$$

2) simplification:

$$f(x) = \frac{7x}{5\sqrt{x}} + \frac{3x^2}{5\sqrt{x}}$$

$$= \frac{7}{5}x^{1/2} + \frac{3}{5}x^{3/2}$$

$$f'(x) = \frac{7}{10}x^{-1/2} + \frac{9}{10}x^{1/2}$$

20) $a(t) = 2\sin^2(t) + 2\cos^2(t)$

1) chain rule:

$$a(t) = 2(\sin(t))^2 + 2(\cos(t))^2$$

$$a'(t) = 4(\sin(t))(\cos(t)) + 4(\cos(t))(-\sin(t))$$

2) trig identity:

$$a(t) = 2(\sin^2(t) + \cos^2(t))$$

$$= 2(1)$$

$$a'(t) = 0$$

21) $m(v) = \arccos(\cos(t))$

1) chain rule:

$$m'(v) = \frac{-1}{\sqrt{1 - (\cos(t))^2}} \cdot (-\sin(t))$$

2) $m(v) = \cancel{\arccos(\cos}(t))$

$$= t$$

$$m'(v) = 1$$

Implicit Differentiation

Use implicit differentiation to calculate $\frac{dy}{dx}$ for the following implicitly defined functions.

22. $\sqrt{xy} = 1 + x^2y$

23. $\cos(x-y) = y\sin(x)$

24. $x^3 + x^2y + 4y^2 = 6$

25. $x^2\sin(y) = \ln(xy)$

22. $\frac{d}{dx}\left(\sqrt{xy}\right) = \frac{d}{dx}(1+x^2y) \rightarrow \frac{d}{dx}\left((xy)^{1/2}\right) = \frac{d}{dx}(1+x^2y) \rightarrow \left(\frac{1}{2}(xy)^{-1/2}\right)\frac{d}{dx}(xy) = \frac{d}{dx}(1) + \frac{d}{dx}(x^2y) \rightarrow$
$\left(\frac{1}{2}(xy)^{-1/2}\right)\left(\left(\frac{d}{dx}(x)\right)(y) + x\left(\frac{d}{dx}(y)\right)\right) = 0 + \left(\frac{d}{dx}(x^2)\right)(y) + (x^2)\left(\frac{d}{dx}(y)\right) \rightarrow$

$\left(\frac{1}{2}(xy)^{-1/2}\right)\left((1)(y) + x\left(\frac{dy}{dx}\right)\right) = (2x)(y) + (x^2)\left(\frac{dy}{dx}\right) \rightarrow \frac{1}{2\sqrt{xy}}\left(y + x\left(\frac{dy}{dx}\right)\right) = 2xy + x^2\left(\frac{dy}{dx}\right) \rightarrow$

$\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}}\frac{dy}{dx} = 2xy + x^2\left(\frac{dy}{dx}\right) \rightarrow \frac{y}{2\sqrt{xy}} - 2xy = x^2\frac{dy}{dx} - \frac{x}{2\sqrt{xy}}\frac{dy}{dx} \rightarrow \frac{y}{2\sqrt{xy}} - 2xy = \left(x^2 - \frac{x}{2\sqrt{xy}}\right)\frac{dy}{dx} \rightarrow$

$\frac{dy}{dx} = \frac{\frac{y}{2\sqrt{xy}} - 2xy}{x^2 - \frac{x}{2\sqrt{xy}}}$

Optional simplify: $\frac{dy}{dx} = \frac{y - 2xy(2\sqrt{xy})}{x^2(2\sqrt{xy}) - x} \rightarrow \frac{dy}{dx} = \frac{y - 4(xy)^{\frac{3}{2}}}{x^2(2\sqrt{xy}) - x}$

23. $\frac{d}{dx}(\cos(x-y)) = \frac{d}{dx}(y\sin(x)) \rightarrow (-\sin(x-y))\left(1 - \frac{dy}{dx}\right) = \frac{dy}{dx}\sin(x) + y\cos(x) \rightarrow$

$(-\sin(x-y)) - (-\sin(x-y))\frac{dy}{dx} = \frac{dy}{dx}\sin(x) + y\cos(x) \rightarrow$

$(-\sin(x-y))\frac{dy}{dx} - \sin(x)\frac{dy}{dx} = (-\sin(x-y)) + y\cos(x) \rightarrow$

$\left((-\sin(x-y)) - \sin(x)\right)\frac{dy}{dx} = (-\sin(x-y)) + y\cos(x) \rightarrow$

$\frac{dy}{dx} = \frac{(-\sin(x-y)) + y\cos(x)}{((-\sin(x-y)) - \sin(x))} \rightarrow \frac{dy}{dx} = \frac{\sin(x-y) - y\cos(x)}{\sin(x-y) + \sin(x)}$

24. $\frac{d}{dx}(x^3 + x^2y + 4y^2) = \frac{d}{dx}(6) \rightarrow 3x^2 + \left(2xy + x^2\frac{dy}{dx}\right) + 8y\frac{dy}{dx} = 0 \rightarrow x^2\frac{dy}{dx} + 8y\frac{dy}{dx} = -3x^2 - 2xy \rightarrow$

$(x^2 + 8y)\frac{dy}{dx} = -3x^2 - 2xy \rightarrow \frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 8y}$

25. $\frac{d}{dx}(x^2\sin(y)) = \frac{d}{dx}(\ln(xy)) \rightarrow (2x)\sin(y) + x^2\cos(y)\frac{dy}{dx} = \frac{1}{xy}\left(y + x\frac{dy}{dx}\right) \rightarrow$

$(2x)\sin(y) + x^2\cos(y)\frac{dy}{dx} = \frac{y}{xy} + \frac{x}{xy}\frac{dy}{dx} \rightarrow x^2\cos(y)\frac{dy}{dx} - \frac{x}{xy}\frac{dy}{dx} = \frac{y}{xy} - (2x)\sin(y) \rightarrow$

$\left(x^2\cos(y) - \frac{x}{xy}\right)\frac{dy}{dx} = \frac{y}{xy} - (2x)\sin(y) \rightarrow \frac{dy}{dx} = \frac{\frac{y}{xy} - (2x)\sin(y)}{x^2\cos(y) - \frac{x}{xy}}$

Optional Simplify: $\frac{dy}{dx} = \frac{\frac{y}{xy} - (2x)\sin(y)}{x^2\cos(y) - \frac{x}{xy}} \rightarrow \frac{dy}{dx} = \frac{\frac{1}{x} - (2x)\sin(y)}{x^2\cos(y) - \frac{1}{y}} \rightarrow \frac{dy}{dx} = \frac{1 - (2x^2)\sin(y)}{x^3\cos(y) - \frac{x}{y}} \rightarrow \frac{dy}{dx} = \frac{y - (2x^2y)\sin(y)}{x^3y\cos(y) - x}$

Logarithmic differentiation.

(26.) Let $y = x^x$ We want to find $\frac{dy}{dx} = y'$

$$\ln y = \ln(x^x)$$

$$\ln y = x \cdot \ln(x)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln(x))$$

$$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln(x) \cdot 1$$

$$\frac{y'}{y} = (\ln(x) + 1)$$

$$y' = y \cdot (\ln(x)+1) \Rightarrow y' = x^x (\ln(x)+1) \blacksquare$$

(27.) $f(x) = \frac{(x+2)^2 e^{100+x^3}}{\sin^7(x)}$

$$\ln(f(x)) = \ln\left[\frac{(x+2)^2 e^{100+x^3}}{\sin^7(x)}\right]$$

$$= \ln(x+2)^2 + \ln(e^{100+x^3}) - \ln(\sin^7(x))$$

$$= 2\ln(x+2) + (100+x^3)\ln e - 7\ln(\sin x)$$

$$= 2\ln(x+2) + (100+x^3) \cdot 1 - 7\ln(\sin x)$$

$$\frac{d(\ln(f(x)))}{dx} = \frac{d}{dx}\left[2\ln(x+2) + (100+x^3) - 7\ln(\sin x)\right]$$

$$\frac{f'(x)}{f(x)} = \frac{2}{(x+2)} + 3x^2 - \frac{7 \cdot \cos x}{\sin x}$$

$$f'(x) = \left[\frac{2}{(x+2)} + 3x^2 - \frac{7\cos x}{\sin x}\right] \cdot f(x)$$

$$= \left[\frac{2}{(x+2)} + 3x^2 - \frac{7\cos x}{\sin x}\right]\left[\frac{(x+2)^2 \cdot e^{100+x^3}}{\sin^7(x)}\right]$$

Proofs

28. Prove the product rule using the limit definition of the derivative.
 - This is given in the text book, section 3.3.

29. Prove that $\frac{d}{dx}[\sin x] = \cos x$ using the limit definition of the derivative.
 - This is given in the text book, section 3.6

30. $\frac{d}{dx}(\tan x) = \sec^2 x$ — text book section 3.6

$$\frac{d}{dx}(\sec x) = \frac{d((\cos x)^{-1})}{dx} \underset{\text{Quotient rule}}{=} \frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x \cdot \cos x}$$

$$= \sec x \cdot \tan x \quad \blacksquare$$

The question asks you to use only the trig-identities, the quotient rule and the derivatives of sine & cosine. However if we had the freedom to use the chain rule we can do it like below.

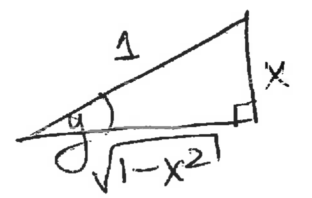
$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left((\cos x)^{-1}\right) \underset{\text{chain rule}}{=} -(\cos x)^{-2}\cdot(-\sin x)$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \sec x \cdot \tan x \quad \blacksquare$$

(31.) $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$ using implicit. dif.

Let $y = \arcsin x. \Rightarrow \sin y = x$



$$\frac{d}{dx}(\sin y) = \frac{d(x)}{dx}$$

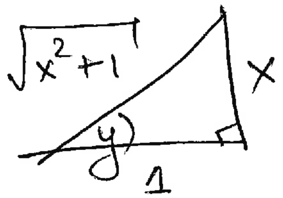
$$\cos y \cdot \frac{dy}{dx} = 1$$

$\cos y = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}} \quad \blacksquare$$

(32) $\frac{d}{dx}[\arctan(x)] = \frac{1}{x^2+1}$ using implicit diff.

Let $y = \arctan x$



$\tan y = x$

$\cos y = \frac{1}{\sqrt{x^2+1}}$

$\frac{d}{dx}(\tan y) = \frac{d(x)}{dx}$

$\sec^2 y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \frac{1}{x^2+1}$ ∎

(33.) $\frac{d}{dx}[b^x] = b^x \ln(b)$ using implicit diff.

Let $y = b^x$. We want to find $\frac{dy}{dx} = y'$

$\ln y = \ln b^x$

$\ln y = x \cdot \ln b$

Note to student: (note that ($\ln b$)) is a constant!)

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln b)$

$\frac{y'}{y} = \ln b \cdot \frac{d}{dx}(x)$

$y' = \ln b \cdot y = \ln b \cdot b^x$ ∎

(34.) Prove that $\frac{d}{dx}[\ln x] = \frac{1}{x}$ for $x > 0$

– This is given in the text book, section 3.9

(35.) Prove that $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$

We know that $f(f^{-1}(x)) = x$

$$\frac{d}{dx}\left[f(f^{-1}(x))\right] = \frac{d(x)}{dx}$$

$$f'(f^{-1}(x)) \, \frac{d(f^{-1}(x))}{dx} = 1.$$

chain rule

$$\frac{d(f^{-1}(x))}{dx} = \frac{1}{f'(f^{-1}(x))}$$

$\blacksquare$

36. If $f(2)=4$ and $f'(2)=7$, determine the derivative of f^{-1} at 4.

Let $f^{-1}(x) = g(x)$.

$$f^{-1}(f(x)) = x$$

$$g(f(x)) = x$$

$$\frac{d}{dx} g(f(x)) = \frac{d}{dx} x$$

$$g'(f(x)) \cdot f'(x) = 1$$

$$g'(f(2)) \cdot f'(2) = 1$$

$$g'(4) \cdot 7 = 1$$

$$\boxed{g'(4) = \frac{1}{7}}$$

37. If $f(x) = \frac{2x-1}{3x+4}$, determine $\frac{d}{dx}[f^{-1}(x)]$ in 2 ways.

(i) $y = \frac{2x-1}{3x+4}$, so to find $f^{-1}(x)$, switch x and y.

$$x = \frac{2y-1}{3y+4}$$

$$3xy + 4x = 2y - 1$$

$$3xy - 2y = -4x - 1$$

$$y(3x-2) = -4x - 1$$

$$y = \frac{-4x-1}{3x-2} = f^{-1}(x)$$

$$\frac{d}{dx}[f^{-1}(x)] = \frac{d}{dx} \frac{-4x-1}{3x-2}$$

$$-4x-1 \qquad 3x-2$$

$$-4 \qquad 3$$

$$\frac{d}{dx}[f^{-1}(x)] = \frac{-12x + 8 + 12x + 3}{(3x-2)^2}$$

$$= \boxed{\frac{11}{(3x-2)^2}}$$

37. (ii) $f(f^{-1}(x)) = x$ $\qquad f = \frac{2x-1}{3x+4}$ $\qquad$ By (i), $f^{-1}(x) = \frac{-4x-1}{3x-2}$

$\frac{d}{dx} f(f^{-1}(x)) = \frac{d}{dx} x$ $\qquad \frac{2x-1}{2} \quad \frac{3x+4}{3}$

$f'(f^{-1}(x)) \cdot \frac{d}{dx} f^{-1}(x) = 1$ $\qquad f'(x) = \frac{6x+8-6x+3}{(3x+4)^2}$

$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$ $\qquad = \frac{11}{(3x+4)^2}$

$$= \frac{1}{\frac{11}{\left(3\left(\frac{-4x-1}{3x-2}\right)+4\right)^2}} = \frac{\left(3\left(\frac{-4x-1}{3x-2}\right)+4\right)^2}{11} = \frac{\left(\frac{-12x-3}{3x-2} + \frac{12x-8}{3x-2}\right)^2}{11}$$

$$= \frac{\left(\frac{-11}{3x-2}\right)^2}{11} = \frac{\frac{121}{(3x-2)^2} \cdot \frac{1}{11}}{11} = \boxed{\frac{11}{(3x-2)^2}}$$

---

38. $g(d) = ab^2 + 3c^3d + 5b^2c^2d^2$

$g'(d) = 0 + 3c^3 + 10b^2c^2d$

$g''(d) = 0 + 0 + 10b^2c^2$

$= \boxed{10b^2c^2}$

---

39. $\frac{dy}{dx} = 5$, $\frac{dx}{dt} = -2$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= 5(-2) = \boxed{-10}$

40. $v(t) = h'(t) = 20 - 10t$

$20 - 10t = 0$

$\boxed{t = 2 \text{ seconds}}$

$h(2) = 10 + 20(2) - 5(4)$

$= 10 + 40 - 20$

$= \boxed{30 \text{ m}}$

---

41. $f(x) = \frac{1}{20}x^5 - \frac{1}{6}x^4 + \frac{1}{6}x^3 + 5x + 1$

$f'(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + 5$

$f''(x) = x^3 - 2x^2 + x$

$x^3 - 2x^2 + x = 0$

$x(x^2 - 2x + 1) = 0$

$x(x-1)(x-1) = 0$

$\boxed{x = 0, 1}$

---

42. Find an equation of the line tangent to $y = x^3$ at $x = 2$.

$y' = 3x^2$

$y'(2) = 3(4) = 12 = m$

$y(2) = 8$

$\boxed{y - 8 = 12(x-2)}$

43. Find an equation of the line tangent to $y = 2e^x$ at $x = 1$.

$y' = 2e^x$

$y'(1) = 2e$

$y(1) = 2e$

$\boxed{y - 2e = 2e(x-1)}$

---

44. Find an equation of the line tangent to $x^{2/3} + y^{2/3} = 4$ at $(-3\sqrt{3}, 1)$

$\frac{d}{dx}\left(x^{2/3} + y^{2/3}\right) = \frac{d}{dx} 4$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$ $\quad x = -3\sqrt{3} = -3^{\frac{3}{2}}$ and $y = 1$

$\frac{2}{3}\left(-3^{\frac{3}{2}}\right)^{-\frac{1}{3}} + \frac{2}{3}\frac{dy}{dx} = 0$

$-\frac{2}{3}3^{-\frac{1}{2}} + \frac{2}{3}\frac{dy}{dx} = 0$

$\frac{2}{3}\frac{dy}{dx} = \frac{2}{3}3^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$

$\boxed{y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})}$

45. The line tangent to g at $x=4$ is $y=3x+10$ implies $g'(4)=3$.

$$h(x) = g(x) + f(x)$$
$$= g(x) + 17 - \sqrt{x}$$

$$h'(x) = g'(x) - \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$h'(4) = g'(4) - \tfrac{1}{2}(4)^{-\frac{1}{2}}$$
$$= 3 - \tfrac{1}{2}\left(\tfrac{1}{2}\right) = \boxed{2\tfrac{3}{4}}$$

---

46. $\ln x - y = 0$

$$\ln x = y$$
$$\frac{d}{dx}\ln x = \frac{d}{dx}y$$
$$\boxed{\frac{1}{x} = \frac{dy}{dx}}$$

Misc #47 – #57

#47.

$$\frac{d}{dy}[\ln(x) - y] = \frac{d}{dy}(0)$$

$$\frac{1}{x}\cdot\frac{dx}{dy} - 1 = 0$$

$$\boxed{\frac{dx}{dy} = x}$$

#48

for continuity:

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} ax^2 - a = a^2 - a$$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} ax + 3x^2 = a + 3$$

$$\therefore\ a^2 - a = a + 3$$

$$a^2 - 2a - 3 = 0$$

$$(a-3)(a+1) = 0$$

$$a = 3 \text{ or } -1$$

for differentiability:

for $f(x) = a^2x - a$, $x \le 1$

$$f'(x) = a^2 \Rightarrow f'(1) = a^2$$

for $f(x) = ax + 3x^2$, $x >$

$$f'(x) = a + 6x \Rightarrow f'(1) = a + 6$$

$\therefore$ at $x = 1$,

$$a^2 = a + 6$$

$$a^2 - a - 6 = 0$$

$$(a-3)(a+2) = 0$$

$$a = 3 \text{ or } -2.$$

$$\boxed{a = 3}$$

#49.

ⓐ $g = fh \Rightarrow g' = f'h + f\cdot h' \Rightarrow g'(2) = f'(2)\cdot h(2) + f(2)\cdot h'(2)$

$$= \frac{f(3) - f(1)}{3-1}\cdot 4 + 4\cdot\frac{f(4) - f(0)}{4-0}$$

$$= \frac{5-3}{2}\cdot 4 + 4\cdot\frac{3-5}{4}$$

$$= 4 - 4\cdot\frac{2}{4}$$

$$= 4 - 2 = 2$$

(b) $k = f \circ h \Rightarrow k' = f'(h) \cdot h' \Rightarrow k'(2) = f'[h(2)] \cdot h'(2)$

$\therefore k'(2) = f'[4] \cdot h'(2)$

$= -2 \cdot \frac{1}{2}$

$= -1$

(c) $m = \frac{f}{h}, \Rightarrow m' = \frac{f' \cdot h - f \cdot h'}{h^2} \Rightarrow m'(2) = \frac{f'(2) \cdot h(2) - f(2) \cdot h'(2)}{(h(2))^2}$

$\therefore m'(2) = \frac{1 \cdot 4 - 4 \cdot \frac{1}{2}}{4^2}$

$= \frac{4-2}{16} = \frac{2}{16} = \frac{1}{8}$

#50

(a) $h'(3) = f'(3) \cdot g(3) + f(3) \cdot g'(3) = 4 \cdot 1 + 2 \cdot 3 = 10$

(b) $k'(3) = \frac{f'(3) \cdot g(3) - f(3) \cdot g'(3)}{(g(3))^2} = \frac{4 \cdot 1 - 2 \cdot 3}{1^2} = -2$

(c) $m'(3) = f'(g(3)) \cdot g'(3) = f'(1) \cdot 3 = 5 \cdot 3 = 15$

#51

$\frac{d}{dx}(x^2 + 2y^2) = \frac{d}{dx}(6)$

$2x + 4y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{4y}$

$= -\frac{x}{2y}$

$\therefore \frac{dy}{dx} = -\frac{x}{2y} = 1$

$x = -2y$

$\therefore (-2y)^2 + 2y^2 = 6$

$6y^2 = 6$

$y^2 = 1$

$y = 1$ or -1

for $y=1$, $x=-2$ $\boxed{(-2,1)}$

for $y=-1$, $x=2$ $\boxed{(2,-1)}$

#52.

$$\frac{d}{dx}[f(3x^2)] = f'(3x^2)\cdot(3x^2)' = \boxed{6x\cdot f'(3x^2)}$$

#53.

$y = \sin(x)$

$y' = \cos(x)$

$y'' = -\sin(x)$

$y''' = -\cos(x)$

$y^{(4)} = \sin(x)$

$y^{(5)} = \cos(x)$

$\Rightarrow$ cycle of 4. $\boxed{y^{(45)} = \cos(x)}$

#54.

$f(x) = xe^x \quad = 0\cdot e^x + xe^x$

$f'(x) = e^x + xe^x \quad = 1\cdot e^x + xe^x$

$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$

$f'''(x) = e^x + e^x + e^x + xe^x = 3e^x + xe^x$

$f^{(4)}(x) = e^x + e^x + e^x + e^x + xe^x = 4e^x + xe^x$

$\vdots$

$\boxed{f^{(n)}(x) = n\cdot e^x + xe^x}$, for $n = 0, 1, 2, \cdots, k, \ldots$

#55 :

$$g(x) = \frac{1}{x} = x^{-1} = 0! \cdot x^{-1}$$

$$g^{(1)}(x) = -x^{-2} = -1!\, x^{-2}$$

$$g^{(2)}(x) = 2x^{-3} = 2!\, x^{-3}$$

$$g^{(3)}(x) = -2 \cdot 3 \cdot x^{-4} = -3!\, x^{-4}$$

$$g^{(4)}(x) = 2 \cdot 3 \cdot 4 \cdot x^{-5} = 4!\, x^{-5}$$

$$\vdots$$

$$g^{(n)}(x) = (-1)^n n!\, x^{-(n+1)}, \text{ for } n = 0, 1, 2, \ldots, k, \ldots$$

#56

$$f'(x) = 5x^4 - 6x + 1$$

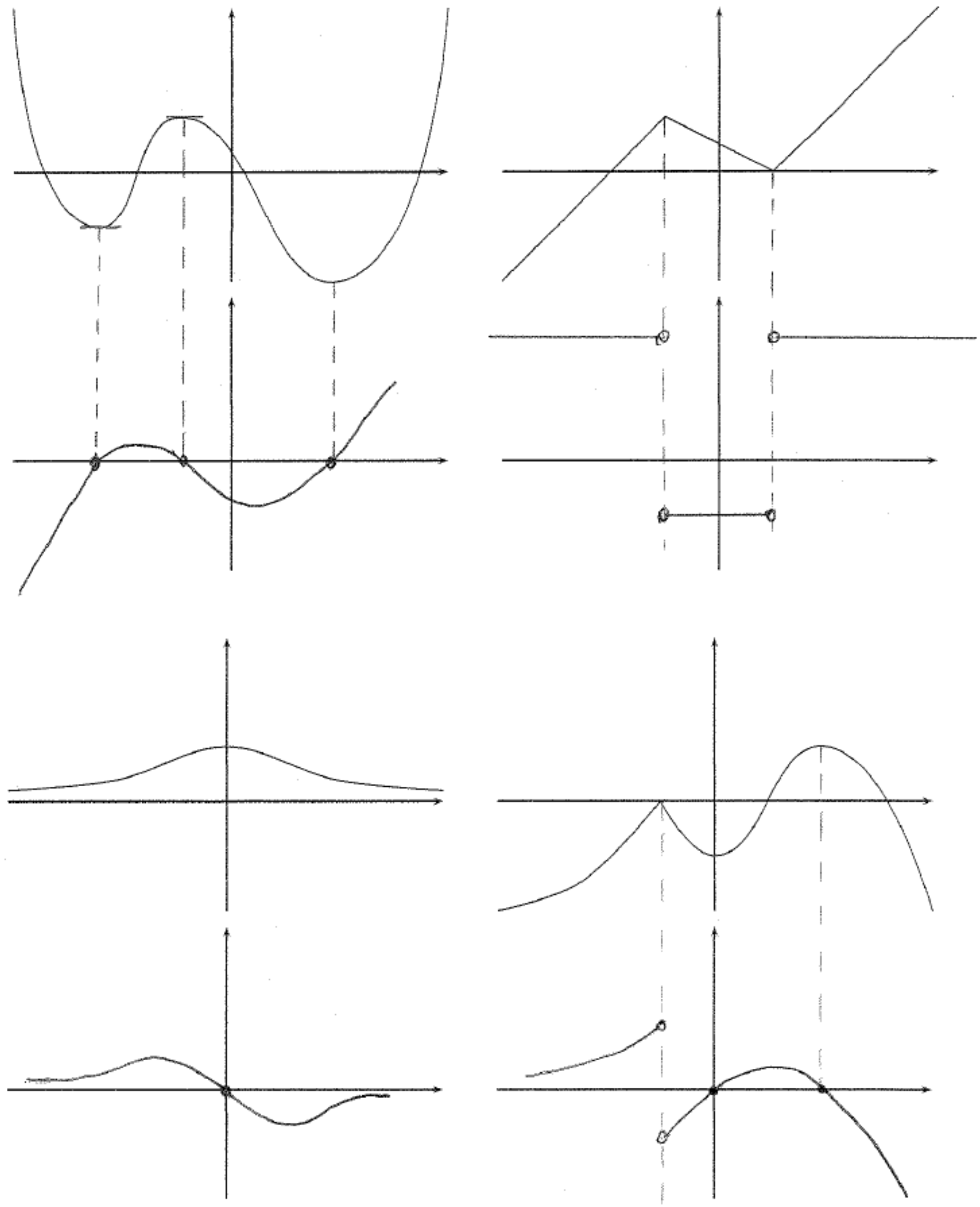
$$\Rightarrow f'(1) = 5 - 6 + 1 = 0$$

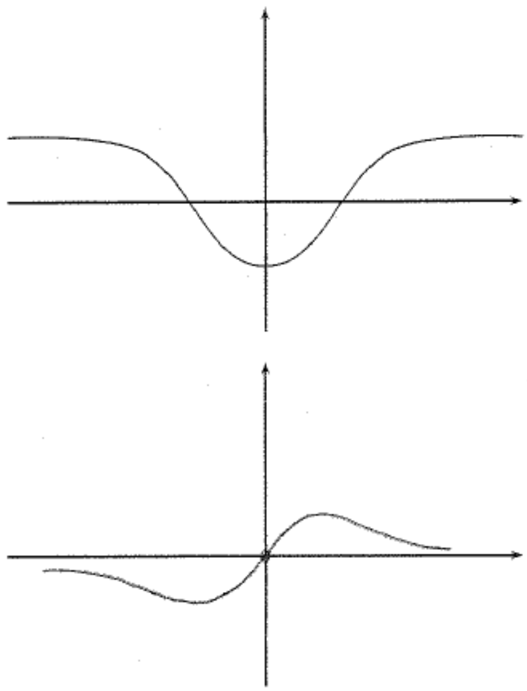
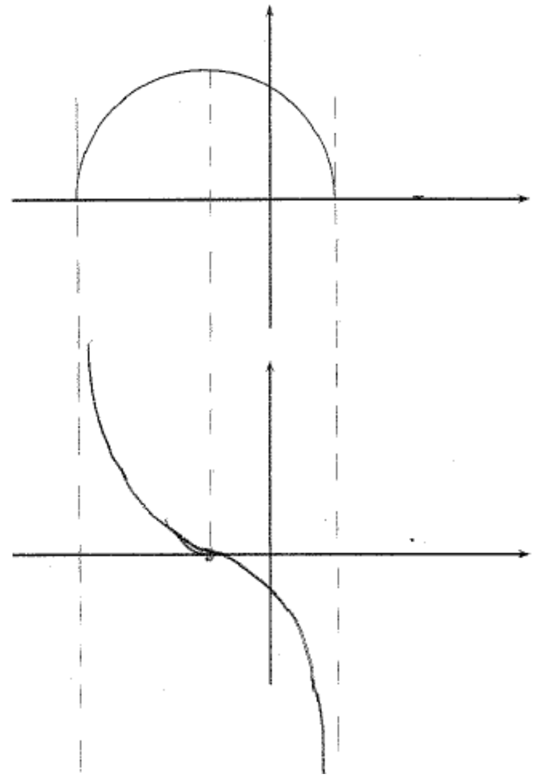
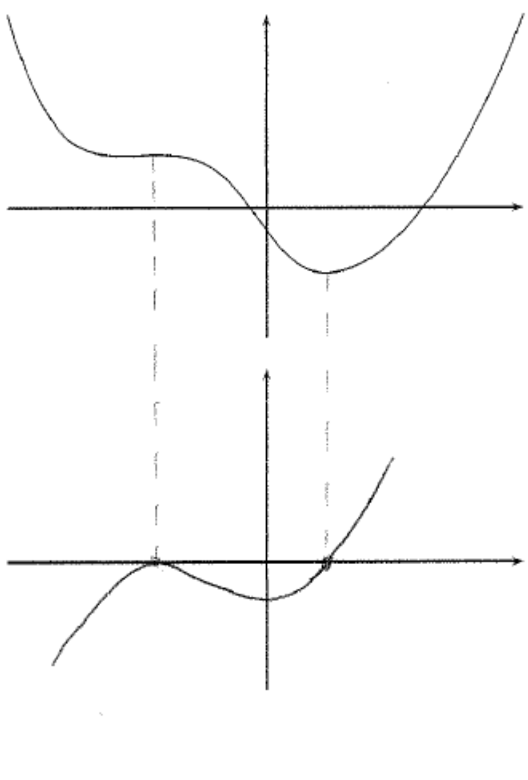
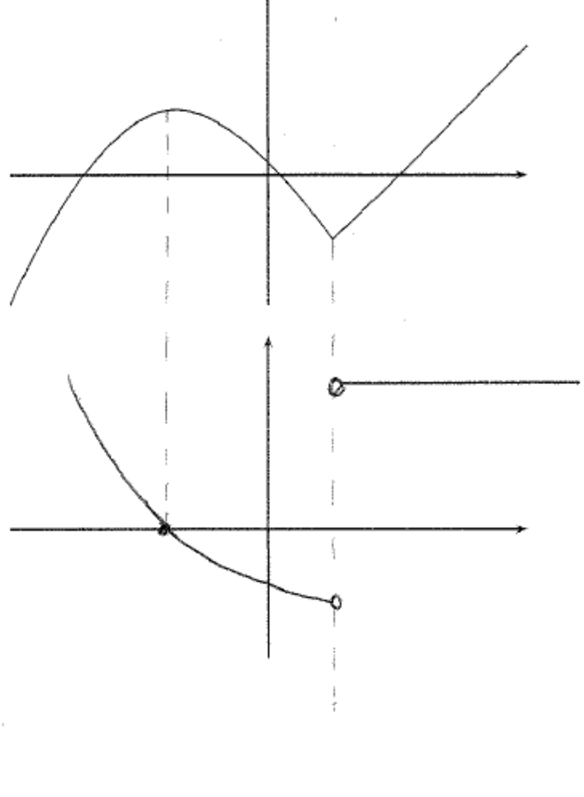
$$\therefore y - f(1) = 0\,(x-1)$$

$$y - 3 = 0$$

$$\boxed{y = 3}$$

57



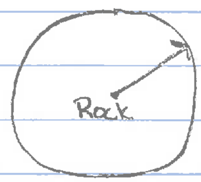
Related Rates

58) $y = x^2 - 1$ Find dy/dt when $x = 2$ and $dx/dt = 3$

$$\frac{dy}{dt} = \frac{d}{dt}(x^2 - 1) = \frac{d}{dt}x^2 - \frac{d}{dt}1$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt} - 0 = 2x\frac{dx}{dt}$$

$$\left.\frac{dy}{dt}\right|_{x=2} = 2(2)(3) = \boxed{12}$$

59) 

Pond

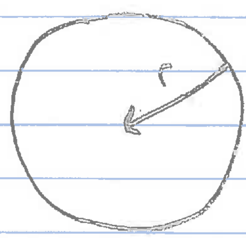
$\frac{dr}{dt} = 12\ \frac{cm}{s}$ At $r = 30cm$, find $\frac{dA}{dt}$

Thing changing wrt time Area, radius, circumference

Use $A = \pi r^2$ b/c asking for dA/dt

$$\frac{dA}{dt} = \frac{d}{dt}\pi r^2 = \pi\frac{d}{dt}r^2 = \pi(2r)\frac{dr}{dt} = 2\pi r\frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{r=30} = 2\pi(30cm)\left(12\ \frac{cm}{s}\right) = \boxed{720\pi\ \frac{cm^2}{s}}$$

60) 

Sphere →

$\frac{dV}{dt} = -3\ \frac{in^3}{s}$ Find $\frac{dD}{dt}$ at $r = 2in$

Changing wrt time
Volume, radius, diameter

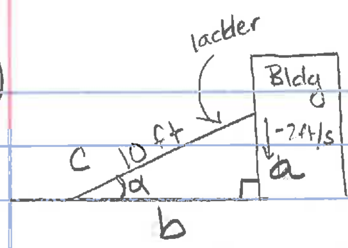
Note: $D = 2r$, so $r = D/2$
when $r = 2in$, $D = 2(2) = 4in$

Use $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{D^3}{8}\right) = \frac{1}{6}\pi D^3$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{1}{6}\pi D^3\right) = \frac{\pi}{6}\frac{d}{dt}(D^3) = \frac{\pi}{6}\cdot 3D^2\frac{dD}{dt} = \frac{\pi}{2}D^2\frac{dD}{dt}$$

At $D = 4$:

$$-3\ \frac{in^3}{s} = \frac{\pi}{2}(4in)^2\frac{dD}{dt} \qquad \frac{dD}{dt} = \left(\frac{-3\ in^3}{s}\right)\left(\frac{2}{\pi}\right)\left(\frac{1}{16in^2}\right) = \boxed{-\frac{3}{8\pi}\ in/sec}$$

(01) 

$c = 10\text{ft}$ (constant) Find $\frac{d\alpha}{dt}$

$\frac{da}{dt} = -2\text{ft/sec}$

What is changing wrt time?

Lengths a & b, the acute angles

Relationship $\sin\alpha = \frac{a}{c} = \frac{a}{10}$

$$\frac{d}{dt}\sin\alpha = \frac{d}{dt}\frac{a}{10}$$

$$\cos\alpha\,\frac{d\alpha}{dt} = \frac{1}{10}\frac{da}{dt}$$

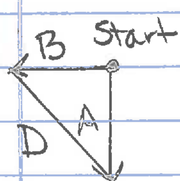
$$\frac{d\alpha}{dt} = \frac{1}{10\cos\alpha}\frac{da}{dt}$$ What is $\cos\alpha$ at $a = 5\text{ft}$

$\cos\alpha = \frac{b}{10}$ $a^2+b^2=c^2$ so $5^2+b^2=10^2$

$b^2 = 100-25 = 75$ $b = \sqrt{75} = 5\sqrt{3}$

$$\therefore \cos\alpha = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

$$\frac{d\alpha}{dt} = \frac{1}{10\left(\frac{\sqrt{3}}{2}\right)}\cdot(-2\text{ ft/sec}) = \boxed{\frac{-2}{5\sqrt{3}}\ \frac{\text{ft}}{\text{sec}}}$$ (moving left)

(02) 

$\frac{dA}{dt} = 60\text{mph}$

$\frac{dB}{dt} = 25\text{mph}$ what is $\frac{dD}{dt}$ at $t = 2\text{hrs}$?

what is changing wrt time?

distances A, B, D

Relationship: $A^2 + B^2 = D^2$

$$\frac{d}{dt}(A^2+B^2) = \frac{d}{dt}D^2$$

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2D\frac{dD}{dt}$$

62 con't

We know dA/dt & dB/dt, but what are A & B at $t = 2$ hours?

$$A = 60\,\frac{\text{mi}}{\text{hr}} \times 2\,\text{hrs} = 120\,\text{mi}$$

$$B = 25\,\frac{\text{mi}}{\text{hr}} \times 2\,\text{hrs} = 50\,\text{mi}$$

$$D^2 = A^2 + B^2$$

$$D^2 = 120^2 + 50^2 = 16{,}900 \qquad D = \sqrt{16900} = 130\,\text{mi}$$

(A 5-12-13 triangle!)

Using $2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2D\frac{dD}{dt}$

$$2(120\,\text{mi})\left(60\,\frac{\text{mi}}{\text{hr}}\right) + 2(50\,\text{mi})\left(25\,\frac{\text{mi}}{\text{hr}}\right) = 2(130\,\text{mi})\frac{dD}{dt}$$

$$14{,}400 + 25{,}000 = 260\, dD/dt$$

$$16900 = 260\, dD/dt$$

$$\frac{dD}{dt} = 16900 \div 260 = \boxed{65\,\text{mph}}$$

ADD'L DERIVATIVE PRACTICE

Power Rule

63) $f(x) = x - x^3 \qquad f'(x) = 1 - 3x^2$

64) $y = 3x^2 - \sqrt{x} + \frac{4}{x} + \pi^2 = 3x^2 - x^{1/2} + 4x^{-1} + \pi^2$

$y' = 6x - \frac{1}{2}x^{-1/2} - 4x^{-2}$

65) $f(x) = \frac{4}{x^2} - \frac{x^2}{4} = 4x^{-2} - \frac{1}{4}x^2 \qquad f'(x) = -8x^{-3} - \frac{1}{2}x$

66) $h(x) = \frac{3}{\sqrt{x}} = 3x^{-1/2} \qquad h'(x) = -\frac{3}{2}x^{-3/2}$

67) $f(x) = x^2 - e^2 \qquad f'(x) = 2x$

68) $g(x) = \sqrt{\sqrt{x}} = (x^{1/2})^{1/2} = x^{1/4} \qquad g'(x) = \frac{1}{4}x^{-3/4}$

Chain Rule

69) $f(x) = (x^2-1)^{10} \qquad f'(x) = 10(x^2-1)^9(2x) = 20x(x^2-1)^9$

70) $f(x) = \sqrt{1+\sqrt{1+2x}}$

There are several choices for the inside & outside functions. My choice:

$\Theta = \sqrt{x} = x^{1/2} \quad I = 1 + \sqrt{1+2x} = 1 + (1+2x)^{1/2}$

$\Theta' = \frac{1}{2}x^{-1/2} \quad I' = 0 + \frac{1}{2}(1+2x)^{-1/2}(2) = (1+2x)^{-1/2}$

$f'(x) = \frac{1}{2}(1+\sqrt{1+2x})^{-1/2}(1+2x)^{-1/2}$

71) $g(x) = (3x^2+3x-6)^{-8} \qquad g'(x) = -8(3x^2+3x-6)^{-9}(6x+3)$

72) $f(x) = \sqrt[4]{9-x} = (9-x)^{1/4} \qquad f'(x) = \frac{1}{4}(9-x)^{-3/4}(-1)$

$= -\frac{1}{4}(9-x)^{-3/4}$

Power, Product, Quotient, Chain Rules

73) $h(x) = (\sqrt{x} - 4)^3 (\sqrt{x} + 4)^5 = (x^{1/2} - 4)^3 (x^{1/2} + 4)^5$

$h'(x) = 3(x^{1/2} - 4)^2 (\frac{1}{2} x^{-1/2})(x^{1/2} + 4)^5 + (x^{1/2} - 4)^3 (5)(x^{1/2} + 4)^4 (\frac{1}{2} x^{-1/2})$

$= \frac{3(x^{1/2} - 4)^2 (x^{1/2} + 4)^5}{2\sqrt{x}} + \frac{5(x^{1/2} - 4)^3 (x^{1/2} + 4)^4}{2\sqrt{x}}$

74) $f(x) = x\sqrt{3x^2 - x} = x(3x^2 - x)^{1/2}$

$f'(x) = (1)(3x^2 - x)^{1/2} + x(\frac{1}{2})(3x^2 - x)^{-1/2}(6x - 1)$

$= \sqrt{3x^2 - x} + \frac{x(6x - 1)}{2\sqrt{3x^2 - x}}$

75) $f(x) = \frac{(5x^2 - 3)(x^2 - 2)}{x^2 + 2} = \frac{5x^4 - 10x^2 - 3x^2 + 6}{x^2 + 2}$

$= \frac{5x^4 - 13x^2 + 6}{x^2 + 2}$ (or you can use the Product Rule plus the Quotient

$f'(x) = \frac{(20x^3 - 26x)(x^2 + 2) - (5x^4 - 13x^2 + 6)(2x)}{(x^2 + 2)^2}$ STOP!

76) $g(x) = \frac{x}{x + 17/x} = \frac{x}{x + 17x^{-1}}$

$g'(x) = \frac{1(x + \frac{17}{x}) - x(1 - 17x^{-2})}{(x + \frac{17}{x})^2}$

77) $f(t) = 3t^2 + 2t$ $f'(t) = 6t + 2$

78) $g(w) = \frac{w^3}{(w+3)^5}$ $g'(w) = \frac{3w^2(w+3)^5 - w^3(5)(w+3)^4(1)}{[(w+3)^5]^2}$

$g'(w) = \frac{3w^2(w+3)^5 - 5w^3(w+3)^4}{(w+3)^{10}}$

OPTIONAL: $g'(w) = \frac{3w^2(w+3) - 5w^3}{(w+3)^6}$

79) $h(s) = (s^{-2})^3 = s^{-6}$ $\quad h'(s) = -6s^{-7}$

80) $f(x) = 5\sqrt{x} = 5x^{1/2}$ $\quad f'(x) = \frac{5}{2}x^{-1/2} = \frac{5}{2\sqrt{x}}$

81) $g(x) = \sqrt[3]{\sqrt[5]{\sqrt{x}}} = ((x^{1/2})^{1/5})^{1/3} = x^{1/30}$ $\quad g'(x) = \frac{1}{30}x^{-29/30}$

82) $m(t) = \sqrt{t^2 - 5t} = (t^2 - 5t)^{1/2}$

$m'(t) = \frac{1}{2}(t^2 - 5t)^{-1/2}(2t - 5)$

83) $g(y) = \sqrt{1 + \sqrt{1 + \sqrt{y}}} = (1 + (1 + y^{1/2})^{1/2})^{1/2}$

$g'(y) = \frac{1}{2}(1 + (1 + y^{1/2}))^{-1/2}(0 + \frac{1}{2}(1 + y^{\frac{1}{2}})^{-\frac{1}{2}}(0 + \frac{1}{2}y^{-1/2}))$

$= \frac{1}{2}(1 + (1 + y^{1/2}))^{-1/2}(\frac{1}{2}(1 + y^{1/2})^{-1/2}(\frac{1}{2}y^{-1/2})$

$= \frac{1}{8}(1 + (1 + y^{1/2}))^{-1/2}(1 + y^{1/2})^{-1/2}(y^{-1/2})$

84) $h(s) = (s+1)^5\sqrt{s-1} = (s+1)^5(s-1)^{1/2}$

$h'(s) = 5(s+1)^4(1)(s-1)^{1/2} + (s+1)^5(\frac{1}{2})(s-1)^{-1/2}(1)$

$= 5(s+1)^4\sqrt{s-1} + \frac{(s+1)^5}{2\sqrt{s-1}}$

85) $f(x) = \frac{2x-1}{\sqrt{x+1}} = \frac{2x-1}{(x+1)^{1/2}}$

$f'(x) = \frac{2(x+1)^{1/2} - (2x-1)(\frac{1}{2})(x+1)^{-1/2}}{((x+1)^{1/2})^2} = \frac{2\sqrt{x+1} - \frac{1}{2}(2x-1)(x+1)^{-1/2}}{x+1}$

OR rewrite as $f(x) = (2x-1)(x+1)^{-1/2}$ and use product Rule.

86) $f(x) = \dfrac{(x+2)^2(3x-4x^5)^{100}}{(8-x)^7}$ I would prefer to use log tricks, but we're not in that section.

$$f'(x) = \frac{[2(x+2)(1)(3x-4x^5)^{100} + (x+2)^2(100)(3x-4x^5)^{99}(3-20x^4)](8-x)^7}{((8-x)^7)^2}$$

$$- \frac{(x+2)^2(3x-4x^5)^{100}(7)(8-x)^6(-1)}{((8-x)^7)^2}$$

$$f'(x) = \frac{[2(x+2)(3x-4x^5)^{100} + 100(x+2)^2(3x-4x^5)^{99}(3-20x^4)](8-x)^7 + 7(x+2)^2(3x-4x^5)^{100}(8-x)^6}{(8-x)^{14}}$$

Phew!

Exponential Functions

87) $f(t) = e^{3t}$ $f'(t) = e^{3t}(3) = 3e^{3t}$

88) $y = t^2 e^{t^3}$ $y' = 2te^{t^3} + t^2 e^{t^3}(3t^2) = 2te^{t^3} + 3t^4 e^{t^3}$

89) $g(z) = \left(\frac{2}{3}\right)^{3z-z^2}$ $g'(z) = \ln\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^{3z-z^2}(3-2z)$

90) $h(k) = 7e^{-5} - 7e^{-5k} + k^2\ln(e^4)$

$h'(k) = 0 - 7(e^{-5k})(-5) + \ln(e^4)(2k)$

$= 35e^{-5k} + 2\ln(e^4)\,k$

91) $i(r) = 2^{4\sqrt{r}} = 2^{4r^{1/2}}$ $i'(r) = \ln(2)(2^{4r^{1/2}})(4 \cdot \frac{1}{2} r^{-1/2})$

$= \dfrac{2\ln(2)2^{4\sqrt{r}}}{\sqrt{r}}$

92) $A(t) = Pe^{rt}$ $A'(t) = Pe^{rt}(r) = Pre^{rt}$

Logarithmic Functions

93) $l(t) = \ln(x^2-1) \qquad l'(t) = \frac{1}{x^2-1}(2x) = \frac{2x}{x^2-1}$

94) $y = x\ln(x) \qquad y' = 1(\ln x) + x\left(\frac{1}{x}\right) = \ln(x)+1$

95) $h(x) = \ln(x^x) = x\ln x \qquad h'(x) = \ln(x)+1$

96) $t(y) = y\ln\left(\frac{1}{y}\right) = y\ln(y^{-1})$

$t'(y) = 1\left(\ln\left(\frac{1}{y}\right)\right) + y\left(\frac{1}{y^{-1}}\right)(-y^{-2}) = \ln\left(\frac{1}{y}\right) - y(y)\left(\frac{1}{y^2}\right)$
$= \ln\left(\frac{1}{y}\right) - 1$

97) $j(x) = \ln\left(\frac{(4x-1)^8(3x^2+14)^7}{\sqrt{x^2-4}}\right) = \ln(4x-1)^8 + \ln(3x^2+14)^7 - \ln(x^2-4)^{1/2}$

$j'(x) = \frac{1}{(4x-1)^8}(8)(4x-1)^7(4) + \frac{1}{(3x^2+14)^7}(7)(3x^2+14)^6(6x) - \frac{1}{(x^2-4)^{1/2}}\left(\frac{1}{2}\right)(x^2-4)^{-1/2}(2x)$

$j'(x) = \frac{32(4x-1)^7}{(4x-1)^8} + \frac{42x(3x^2+14)^6}{(3x^2+14)^7} - \frac{x}{x^2-4}$

98) $K(s) = \log_2((5s^8-11)^3)$ Use change of base formula

$K(s) = \frac{\ln((5s^8-11)^3)}{\ln 2} = \frac{1}{\ln 2}\left(\ln((5s^8-11)^3)\right)$

$K'(s) = \frac{1}{\ln 2}\left(\frac{1}{(5s^8-11)^3}\right)\left(3(5s^8-11)^2(40s^7)\right) = \frac{120s^7}{\ln 2(5s^8-11)}$

Trig, Inverse Trig, Inverse Functions

99) $b(t) = 4\ln(5\cos(t))$

$b'(t) = 4\dfrac{1}{\cancel{5}\cos(t)}(\cancel{5}(-\sin(t))) = -4\tan(t)$

100) $c(u) = \cos(\sin(u))$

$c'(u) = -\sin(\sin(u))(\cos(u))$

101) $f(w) = \tan(w^2+1)$ $\quad f'(w) = \sec^2(w^2+1)(2w)$

102) $g(v) = \arcsin(\cos(v)) + \cos(\arcsin(v))$

$g'(v) = \dfrac{1}{\sqrt{1-\cos^2(v)}}(-\sin(v)) + -\sin(\arcsin(v))\left(\dfrac{1}{\sqrt{1-v^2}}\right)$

$= \dfrac{-\sin(v)}{\sqrt{1-\cos^2(v)}} - \dfrac{\sin(\arcsin(v))}{\sqrt{1-v^2}}$

103) $h(y) = y^2\arctan(4y)$

$h'(y) = 2y\arctan(4y) + y^2\left(\dfrac{1}{(4y)^2+1}\right)(4)$

$h'(y) = 2y\arctan(4y) + \dfrac{4y^2}{16y^2+1}$

104) $i(z) = \sec^7(2z) = (\sec(2z))^7$

$i'(z) = 7(\sec(2z))^6(\sec(2z)\tan(2z))(2)$

$= 14(\sec(2z))^6(\sec(2z)\tan(2z))$

Linear Approximation

1) Local linearization: $\ell_a(x) = f(a) + f'(a)(x-a)$

a) let $a = \pi/6$ $\qquad f(a) = \tan(\pi/6) = 1/\sqrt{3}$

$f'(x) = \sec^2(x) \qquad f'(a) = \sec^2(\pi/6) = 4/3$

$\ell_{\pi/6}(x) = 1/\sqrt{3} + 4/3\,(x - \pi/6)$

b) let $a = 0 \qquad f(a) = \ln(\cancel{e^0} \overset{2}{+} \cancel{e^{2(0)}}) = \ln(2)$

$f'(x) = \frac{1}{e^x + e^{2x}}(e^x + 2e^{2x}) \qquad f'(0) = \frac{1}{\cancel{e^0 + e^{2(0)}}}(\cancel{e^0} \overset{3}{+} \cancel{2e^{2(0)}}) = \frac{3}{2}$

$\ell_0(x) = \ln(2) + \frac{3}{2}(x - 0)$

2) $\ell_2(x) = 3x - 9 \Rightarrow a = 2, \quad g'(2) = 3$ (slope of line)

$h'(2) = f'(g(2)) \cdot g'(2) \qquad f'(x) = 4e^{4x}$

$= 4e^{4(-3)} \cdot 3 \qquad g(2) = 3(2) - 9 = -3$

$= 12e^{-12}$

3) Approximations using local linearization

a) let $a = 1$ (since we know $\ln(1)$)

$f(x) = \ln(x)$

$f'(a) = \frac{1}{1} = 1$

$\Rightarrow \ell_1(x) = f(1) + f'(1)(x-1)$

$= \underset{0}{\cancel{\ln(1)}} + 1(x-1) = x - 1$, so $\ln(0.9) \approx 0.9 - 1 = -0.1$

b) let $a = 100$ (since we know $\sqrt{100}$)

$f(x) = \sqrt{x} \qquad f'(a) = \frac{1}{2}(100)^{-1/2} = \frac{1}{2\sqrt{100}} = \frac{1}{20} \qquad f(a) = \sqrt{100} = 10$

$\Rightarrow \ell_{100}(x) = f(100) + f'(100)(x - 100)$

$= 10 + \frac{1}{20}(x - 100)$

So $\sqrt{101} \approx 10 + \frac{1}{20}(101 - 100) = 10 + \frac{1}{20}$ or 10.05