Quiz 4

Your Name:

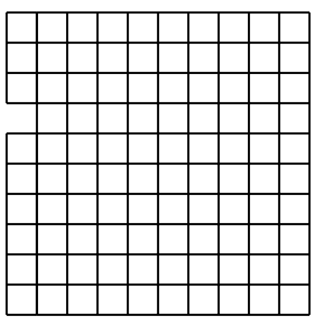
Instructions

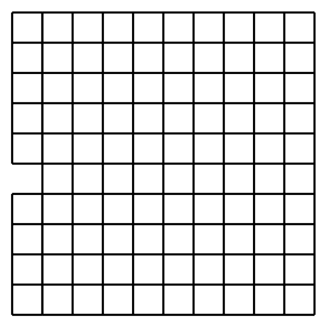
This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

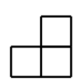
Complete **two** of the following problems.

A1. Tile each of the grids below with trominoes that consist of 3 squares in a line. If a tiling is not possible, explain way.

(a) 

(b) 

A2. Prove that every rectangle of size $m \times n$ such that mn is even and divisible by 3 can be tiled using the following tromino.



A3. Suppose you have $n \geq 2$ coins, all identical in appearance and weight except for one that is either heavier or lighter than the other $n-1$ coins. Suppose our goal is to identify the counterfeit coin with a two-pan scale using the minimal number of weighings and to be able to determine if the counterfeit coin is heavier or lighter than the remaining coins. Let k denote the number of weighing used to detect the counterfeit coin. For part of Problem 53, we determined the following necessary condition:

$$n \leq \frac{3^k - 1}{2}.$$

Determine which numbers of coins we handle in at most 2 weighings. You must justify that your answer is correct.

Part B

Complete **two** of the following problems.

B1. Suppose you have 5 coins, all identical in appearance and weight except for one that is either heavier or lighter than the other 4 coins. What is the minimum number of weighings one must do with a two-pan scale in order to identify the counterfeit and determine whether the counterfeit is heavier or lighter than the others? Describe your method in detail.

B2. Suppose there are two bags of candy containing 8 pieces and 6 pieces, respectively. You and your friend are going to play a game to see who gets all 14 pieces of candy. Here are the rules for the game:

(a) You and your friend will alternate removing pieces of candy from the bags. Let's assume that you go first.

(b) On each turn, the designated player selects a bag that still has candy in it and then removes at least one piece of candy. The designated player can only remove candy from a single bag and he/she must remove at least one piece.

(c) The winner is the one that removes all the candy from the last remaining bag.

Does one of you have a guaranteed winning strategy? If so, describe that strategy.

B3. Consider the light-up squares problem that we encountered in Problems 31 and 32 and on the last quiz. In class we proved that it is not possible for a starting configuration with fewer than n initial lit squares to result in the entire $n \times n$ board being lit up. However, it is possible if we start with n squares lit up (e.g., start with all the squares on one of the long diagonals lit up). How many configurations of 3 lit up squares will result in the entire 3×3 board being lit up?