Quiz 1

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized.* Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Imagine a hallway with 1000 doors numbered consecutively 1 through 1000. Suppose all of the doors are closed to start with. Then some dude with nothing better to do walks down the hallway and opens all of the doors. Because the dude is still bored, he decides to close every other door starting with door number 2. Then he walks down the hall and changes (i.e., if open, he closes it; if closed, he opens it) every third door starting with door 3. Then he walks down the hall and changes every fourth door starting with door 4. He continues this way, making a total of 1000 passes down the hallway, so that on the 1000th pass, he changes door 1000. At the end of this process, which doors are open and which doors are closed? You must justify your answer.

A2. Imagine you have 49 pebbles, each occupying one square on a 7 by 7 chess board. Suppose that each pebble must move to an adjacent square by only moving up, down, left, or right. If this is possible, describe a solution. If this is impossible, explain why.

A3. I have 10 sticks in my bag. The length of each stick is an integer. No matter which 3 sticks I try to use, I cannot make a triangle out of those sticks. What is the minimum length of the longest stick? You must justify your answer.

Part B

Complete **two** of the following problems.

B1. Imagine you have 10 pebbles, each occupying one square of the shape below. Determine whether each of the following scenarios is possible. If so, describe a solution. If a scenario is impossible, explain why. If you choose to do this problem, you need to complete both parts.

  (a) Suppose that each pebble simultaneously must move to an adjacent square by only moving up, down, left, or right. Two pebbles may swap positions, but this is not necessary.

  (b) Now suppose that all but one pebble (your choice which one) must move to an adjacent square by only moving up, down, left, or right.
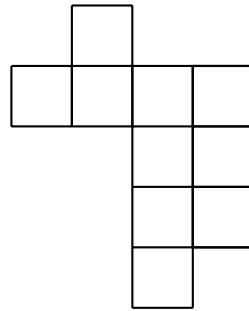
B2. Describe where on Earth from which you can travel one mile west, then one mile south, and then one mile north and arrive at your original location. There is more than one such location. Find them all.

B3. How many ways can 42 be written as the sum of 8 different positive integers?