Quiz 5

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. We have the following information about three integers:

(a) Their product is an integer;

(b) Their product is a prime;

(c) One of them is the average of the other two.

What are these numbers? *Note*: You need to find all such triples and show that there are no others.

A2. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses? *Note:* You do not need to argue that your method is optimal, but you do need to carefully explain each step in your algorithm.

A3. Alice and Brenda both ran in a 100-meter race. When Alice crossed the finish line, Brenda was 10 meters behind her. Assuming the girls run the same rate, how many meters behind Brenda should Alice start in order for them to finish in a tie? *Note:* You must show sufficient work to justify your answer.

Part B

Complete **two** of the following problems.

B1. A signed permutation of the numbers 1 through n is a fixed arrangement of the numbers 1 through n, where each number can be either be positive or negative. For example, $(-2, 1, -4, 5, 3)$ is a signed permutation of the numbers 1 through 5. A *reversal* of a signed permutation swaps the order of a consecutive subsequence of numbers while changing the sign of each number in the subsequence. Performing a reversal to a signed permutation results in a new signed permutation. For example, if we perform a reversal on the second, third, and fourth entries in $(-2, 1, -4, 5, 3)$, we obtain $(-2, -5, 4, -1, 3)$. The *reversal distance* of a signed permutation of 1 through n is the minimum number of reversals required to transform the given signed permutation into $(1, 2, \ldots, n)$. It turns out that $(2, -3, -1)$ has reversal distance 3. Find another signed permutation of 1 through 3 that has reversal distance 3. Demonstrate the reversals needed to transform your permutation into $(1, 2, 3)$.

 Written by D.C. Ernst

B2. If $b = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + 2\sqrt{\cdots}}}}$ is a number, what number is it?

B3. Suppose we must place the letters A, B, C, D, E into the grid below, one per box, so that each row, each column, and each of the two long diagonals contain one of each letter. How many ways are there to fill out the grid and satisfy these conditions? You must justify your answer.

A				B
	B			
		C		
D				E

