# Quiz 6

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. The figure below shows an equilateral triangle ABC with an inscribed semicircle of radius R that is tangent to sides AB and AC, and inscribed circle of radius r that is tangent to the triangle and the semicircle. Find the value of r/R.
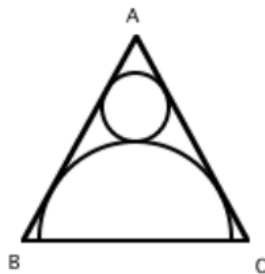

A2. Consider the regular hexagon $ABCDEF$. Let X be the midpoint of CD and let Y be the midpoint of DE. Let Z be the common point of AX and BY. Which polygon has larger area, ABZ or $DXZY$?

A3. Show that in any group of 6 students there are 3 students who know each other or 3 students who do not know each other.

Part B

Complete **two** of the following problems.

B1. A frog jumps along the number line. It starts at 0, chooses which direction to start jumping, and every second it jumps n units in the chosen direction. Once the frog commits to a direction, it always jumps in that direction. Moreover, the jump size is always the same. We want to catch the frog. It's dark, we can't see the frog, we do not know what direction the frog started jumping in, and we don't know what n is. At any given second, we are allowed to choose an integer and search there. If the frog is on that integer, we catch it; if not, we have to try again. How can we catch the frog? We need to know which integer to check at each second.

B2. Cut the following shape into 4 **identical** pieces that can be re-assembled to form a square with the same area. The resulting square cannot have any holes in it.
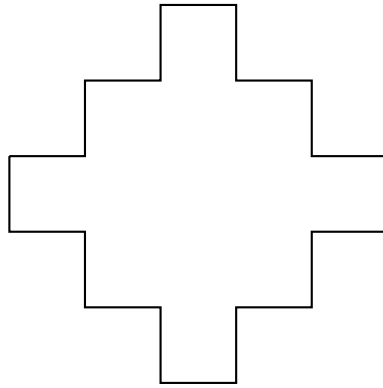
B3. A town of Smurfs consists of 24 blue, 8 pink, and 16 purple individuals. When two Smurts of different colors shake hands, they both change their colors to the third color. Is it possible that all Smurfs in the town eventually have the same color?