## Quiz 5

## Your Name:

## Instructions

This quiz consists of two parts. In each part complete two problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be well-written, neat, and organized. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

## Part A

Complete two of the following problems.
A1. Suppose we must place the letters $A, B, C, D, E$ into the grid below, one per box, so that each row, each column, and each of the two long diagonals contain one of each letter. How many ways are there to fill out the grid and satisfy these conditions? You must justify your answer.

| A |  |  |  | $B$ |
| :---: | :---: | :---: | :---: | :---: |
|  | B |  |  |  |
|  |  | $C$ |  |  |
|  |  |  |  |  |
| D |  |  |  | $E$ |

A2. Show that in any group of 6 students there are 3 students who know each other or 3 students who do not know each other.

A3. During a class period students used their cell phones once. In fact, for any two students there was a time when both of the students used their phones. Show that there was a time when nobody listened to the instructor.

## Part B

Complete two of the following problems.
B1. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type $E$ are there in the puzzle? If it's not possible, explain why. You may assume that the puzzle is solvable. Hint: 253 is divisible by 11 .


B2. Cut the following shape into 4 identical pieces that can be re-assembled to form a square with the same area.


B3. Consider the following dialogue.
William: I have three children.
Harry: What are their ages?
William: The product of their ages is 36 .
Harry: I still don't know their ages.
William: The sum of their ages is your apartment number.
Harry: I still don't know their ages!
William: The oldest plays football.
Harry: Now I know their ages.
What are the ages of William's children?

