Quiz 7

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Suppose you have $n \geq 2$ coins, all identical in appearance and weight except for one that is either heavier or lighter than the other $n-1$ coins. Suppose our goal is to identify the counterfeit coin with a two-pan scale using the minimal number of weighings. Let k denote the number of weighing used to detect the counterfeit coin. For part of Problem 87, we determined the following necessary condition:

$$n \leq \frac{3^k - 1}{2}.$$

Determine which numbers of coins we handle in at most 2 weighings. You must justify that your answer is correct.

A2. Suppose we draw n lines in the plane that have the maximum number of unique intersections. This partitions the plane into disjoint regions (some of which are polygons with finite area and some are not). Suppose we color each of the regions so that no two adjacent regions (i.e., share a common edge) have the same color. What is the fewest colors we could use to accomplish this? Justify your answer.

A3. Prove that every natural number can be written as the sum of distinct powers of two.

Part B

Complete **two** of the following problems.

B1. Two prisoners are locked away in two separate towers, say North Tower and South Tower, and each tower has its own prison guard. Each morning, the respective guards toss a fair coin and then radio the guard in the other tower and report the outcome (heads or tails) of their coin toss. The guard then shows the prisoner in his/her respective tower the outcome of the coin toss in the opposite tower. At this point, each prisoner must guess the outcome of the coin toss that occurred in his/her tower. If at least one of the prisoners guesses correctly, then the prisoners survive another day. If both guess incorrectly, then both will be executed. Is there a strategy that the prisoners can implement that will ensure their survival (until they die of old age in prison) or are they doomed to eventually guess incorrectly and perish? You may assume that prior to being permanently locked up, the prisoners had a few minutes to concoct a plan.

B2. The inhabitants of planet Nuggetron are obsessed with a game called Nuggetto, which is played by two teams at a time. The ranking of a Nuggetto team is based upon the following rules:

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(1) Team X's ranking is the sum of the rankings of the teams that beat Team X.
(2) Team X's ranking is divided evenly by the number of teams that Team X beat.

Suppose Nuggetron has four Nuggetto teams: A, B, C, and D. Suppose that over the course of the season, the following wins occurred:

- Team A beat Teams C and D,
- Team B beat Team A,
- Team C beat Teams A, B, and D,
- Team D beat Team B.

Which team or teams have the highest Nuggetto ranking? You must provide sufficient justification.

B3. A soul swapping machine swaps the souls inside two bodies placed in the machine. Soon after the invention of the machine an unforeseen limitation is discovered: swapping only works on a pair of bodies once. Souls get more and more homesick as they spend time in another body and if a soul is not returned to its original body after a few days, it will kill its current host. Bart (B), Lisa (L), Homer (H), Marge (M), and Ned (N) were involved in a soul-swapping bonanza that resulted in Bart's soul being Lisa's body, Lisa's soul being in Homer's body, Homer's soul being in Marge's body, Marge's soul in Ned's body, and Ned's soul being in Bart's body. Thankfully, Krusty the Clown (K) and Santa's Little Helper (S) never utilized the machine and are available to help put everyone's soul back in the appropriate body. Find a way to return all the souls to their respective owners. Your method must guarantee that pair of bodies never sat in the machine more than once.

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