Quiz 3

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Four red ants and two black ants are walking along the edge of a one meter stick. The four red ants, called Albert, Bart, Debbie, and Edith, are all walking from left to right, and the two black ants, Cindy and Fred, are walking from right to left. The ants always walk at exactly one centimeter per second. Whenever they bump into another ant, they immediately turn around and walk in the other direction. And whenever they get to the end of a stick, they fall off. Albert starts at the left hand end of the stick, while Bart starts 20.2 cm from the left, Debbie is at 38.7cm, Edith is at 64.9cm and Fred is at 81.8cm. Cindy's position is not known—all we know is that he starts somewhere between Bart and Debbie. Which ant is the last to fall off the stick? And how long will it be before he or she does fall off?

A2. There is a plate of n cookies. You and your friend are going to take turns taking either 1 or 2 cookies from the plate. However, it is a faux pas to take the last cookie, so you want to make sure that you do not take the last cookie. For which n does the first player have a guaranteed winning strategy? You must justify your answer.

A3. Rufus and Dufus are identical twins. They are each independently given the same 4-digit number. Rufus takes the number and converts it from decimal (base 10) to base 4, and writes down the 6-digit result. Dufus simply writes the first and last digits of the number followed by the number in its entirety. Rufus is shocked to discover that Dufus has written down exactly the same number has him. What was the original number? In other words, if the original number was $xyzw$, which number $xyzw$, when converted from decimal to base 4 becomes $xwxyzw$?

Part B

Complete **two** of the following problems.

B1. Let t_n denote the nth triangular number. Find both an algebraic proof and a visual proof of the following fact.

For all $a, b \in \mathbb{N}$, $t_{a+b} = t_a + t_b + ab$.

B2. A certain store sells a product called widgets in boxes of $7, 9$, and 11. A number n is called *widgetable* if one can buy exactly n widgets by buying some number of boxes. What is the largest non-widgetable number?

B3. In this problem, we will explore a modified version of the Sylver Coinage Game. In the new version of the game, a fixed positive integer $n \geq 3$ is agreed upon in advance. Then 2 players, A and B, alternately name positive integers from the set $\{1, 2, \ldots, n\}$ that are not the sum of nonnegative multiples of previously named numbers among $\{1, 2, \ldots, n\}$. The person who is forced to name 1 is the loser! Here is a sample game between A and B using the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (i.e., $n = 10$):

(a) A opens with 4. Now neither player can name $4, 8$.

(b) B names 5. Neither player can name $4, 5, 8, 9, 10$.

(c) A names 6. Neither player can name $4, 5, 6, 8, 9, 10$.

(d) B names 3. Neither player can name $3, 4, 5, 6, 7, 8, 9, 10$.

(e) A names 2. Neither player can name $2, 3, 4, 5, 6, 7, 8, 9, 10$.

(f) B is forced to name 1 and loses.

Suppose player A always goes first. Argue that if there exists an n such that player B is guaranteed to win on the set $\{1, 2, \ldots, n\}$ as long as he or she plays intelligently, then player A is guaranteed to win on the set $\{1, 2, \ldots, n, n + 1\}$ as long as he or she plays intelligently. Your argument should describe a strategy for player A.

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