Quiz 4

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

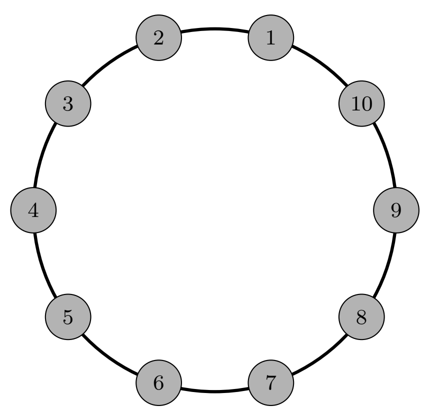
Part A

Complete **two** of the following problems.

A1. The hyper drive of a Star Base allows it to jump from coordinates (a, b) to either coordinates $(a, a+b)$ or to coordinates $(a + b, b)$. If the Star Bases starts at $(1, 0)$, which points in the plane can it get to by using the hyper drive? Justify your answer.

A2. Suppose you randomly pick 3 distinct points on a circle. What is the probability that the center of the circle lies in the interior of the triangle formed by these 3 points?

A3. Ten people form a circle. Each picks a number and tells it to the two neighbors adjacent to him/her in the circle. Then each person computes and announces the average of the numbers of his/her two neighbors. The figure shows the average announced by each person. What is the number picked by the person who announced 6?



Part B

Complete **two** of the following problems.

B1. Alice and Brenda both ran in a 100-meter race. When Alice crossed the finish line, Brenda was 10 meters behind her. The two decide to race each other again. Assuming the girls run the same rate, how many meters behind Brenda should Alice start in order for them to finish in a tie?

B2. Suppose you have 9 coins, all identical in appearance and weight except for one that **we know is heavier** than the other 8 coins. Is it possible to detect the counterfeit coin in at most two weighings with a two-pan scale? If so, describe an algorithm and explain why it works. If this is impossible, explain why.

B3. Determine whether we can obtain every sequence of length 3 in the Circle-Dot system. You must justify your answer. You do not need to reproduce the proofs for sequences we've already obtained, but you should cite the appropriate theorem or proof.

Circle-Dot

Circle-Dot begins with two words; called axioms. Using the two axioms and three rules of inference, we can create new Circle-Dot words, which are theorems in the Circle-Dot System. The process of creating Circle-Dot words using the axioms and rules of inference are proofs in the system. Below are the axioms for Circle-Dot. Note that $\circ$ and $\bullet$ are valid symbols in the system while w and v are variables that stand for any nonempty sequence of $\circ$'s and $\bullet$'s.

Axiom A. $\circ\bullet$

Axiom B. $\bullet\circ$

At any time in your proof, you may quote an axiom. Below are the rules for generating new statements from known statements.

Rule 1. Given wv and vw, conclude w

Rule 2. Given w and v, conclude $w \bullet v$

Rule 3. Given $wv\bullet$, conclude $w\circ$

In class, we proved the following theorems.

Theorem C. $\bullet$

Theorem D. $\circ$

Theorem E. $\bullet \bullet \bullet$

Theorem F. $\bullet \bullet \circ$

Theorem G. $\bullet \circ \circ$

Theorem H. $\circ \bullet \bullet\circ$

Theorem I. $\circ \circ \circ\circ$

Theorem J. $\bullet \circ \bullet$

Theorem K. $\bullet \circ \circ\circ$