Quiz 6

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Recent archaeological work on Mars discovered a site containing a pile of white spheres, each about the size of a tennis ball. A plaque near the mound states that each sphere contains a jewel that come in many different colors while strictly more than half of the spheres contain jewels of the same color. When two spheres are brought together, they both glow white if their internal jewels are the same color; otherwise, no glow. Argue that you can find a sphere that you are certain holds a jewel of the majority color in at most 3 tests if total the number of spheres is 6.

A2. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses? Be sure to explain your reasoning.

A3. Show that in any group of 6 students there are 3 students who know each other or 3 students who do not know each other.

Part B

Complete **two** of the following problems.

B1. Sally was given a set of 5 cards numbered 1 to 5 and Peter was also given a set of 5 cards numbered 1 to 5. They were then blindfolded and told to pick a card from their respective sets. The sum of the numbers from the two cards was told only to Sally and the product of the numbers was told only to Peter. They were then told to guess the two numbers. Below is what each of them said:

Peter: "I do not know the two numbers."

Sally: "Now I know the two numbers."

Peter: "I still don't know the two numbers."

Sally: "Let me help you. The number I was told is larger than the number you were told."

Peter: "Now I know the two numbers."

What are the two numbers?

Written by D.C. Ernst

B2. Two prisoners are locked away in two separate towers, say North Tower and South Tower, and each tower has its own prison guard. Each morning, the respective guards toss a fair coin and then radio the guard in the other tower and report the outcome (heads or tails) of their coin toss. The guard then shows the prisoner in his/her respective tower the outcome of the coin toss in the opposite tower. At this point, each prisoner must guess the outcome of the coin toss that occurred in his/her tower. If at least one of the prisoners guesses correctly, then the prisoners survive another day. If both guess incorrectly, then both will be executed. Is there a strategy that the prisoners can implement that will ensure their survival (until they die of old age in prison) or are they doomed to eventually guess incorrectly and perish? You may assume that prior to being permanently locked up, the prisoners had a few minutes to concoct a plan.

B3. A week before Thanksgiving, a sly turkey is hiding from a family that wants to cook it for the holiday dinner. There are five boxes in a row (numbered 1, 2, 3, 4, 5), and the turkey hides in one of these boxes. Each night, the turkey moves one box to the left or right, hiding in an adjacent box the next day. Each morning, the family can look in one box to try to find the turkey. How can the family guarantee they will find the turkey before Thanksgiving dinner? *Hint:* See if you can first sort out the case when the turkey is in an even-numbered box on the first day.

