Problem 4.116. What are the possible orders for elements in S_6 and A_6? What about S_7 and A_7?

$|S_6| = 6! = 720$

Lagrange's Thm says that any divisor of 720 is a possible order, but certainly not all of these divisors actually occur as orders.

Possible disjoint cycle types in S_6 together w/ their orders:

* $|(1)| = 1$

  $|(ab)| = 2$

* $|(ab)(cd)| = \text{lcm}(2,2) = 2$

  $|(ab)(cd)(ef)| = \text{lcm}(2,2,2) = 2$

* $|(abc)| = 3$

  $|(abc)(de)| = \text{lcm}(3,2) = 6$

* $|(abc)(def)| = \text{lcm}(3,3) = 3$

$|(abcd)| = 4$

* $|(abcd)(ef)| = \text{lcm}(4,2) = 4$

* $|(abcde)| = 5$

$|(abcdef)| = 6$

So, we have elmts of order $1, 2, 3, 4, 5, 6$

Now, let's consider A_6, which consists of the even permutations in S_6.

I've starred the cycle types above w/ pink that yield even perms.

Thus, we have elmts of order $1, 2, 3, 4, 5$ in A_6.

Important: Don't read into this example too much. In S_n, A_n in general the orders aren't always consecutive like above.