Exam 2 (Take-Home Portion)

Your Name:

Names of Any Collaborators:

Instructions

This portion of Exam 2 is worth a total of 22 points and is worth 30% of your overall score on Exam 2. This take-home exam is due at the beginning of class on **Friday, April 19**. Your overall score on Exam 2 is worth 20% of your overall grade. Good luck and have fun!

I expect your solutions to be *well-written*, *neat*, *and organized*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

Feel free to type up your final version. The LATEX source file of this exam is also available if you are interested in typing up your solutions using LATEX. I'll gladly help you do this if you'd like.

The simple rules for the exam are:

1. You may freely use any theorems or problems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem X or Problem Y, then you should say so.

2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.

3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.

4. You are **NOT** allowed to copy someone else's work.

5. You are **NOT** allowed to let someone else copy your work.

6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.**

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

All of the problems on this exam require content from Chapter 8. In particular, you will need to utilize the definition of *continuous function* given on page 102. If you are asked to prove a theorem from Chapter 8, you are allowed to utilize any earlier result in Chapter 8 even if you have not proven the earlier result. For example, you may use Theorem 8.1 to prove Theorem 8.5.

1. (2 points) Define $f : \mathbb{R} \to \mathbb{R}$ via

$$f(x) = \begin{cases} x^2, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0. \end{cases}$$

   Verify that f is not continuous using the definition provided on page 102.

2. (2 points each) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Answer each of the following questions and *briefly* justify your answer.

   (a) Can you identify a topology on either of the domain or codomain of f such that f is guaranteed to be continuous?

   (b) Can you identify a topology on either of the domain or codomain of f such that f is guaranteed to *not* be continuous?

3. (4 points) Prove Theorem 8.1.

4. (4 points) Prove Theorem 8.5

5. (4 points each) Prove **two** of Theorems 8.7, 8.8, 8.15, 8.18.