Final Exam (Take-Home Portion)

Your Name:

Names of Any Collaborators:

Instructions

This portion of the Final Exam is worth a total of 16 points and is worth 30% of your overall score on the Final Exam. This take-home exam is due by 5ᴘᴍ **Thursday, May 9**. Your overall score on the Final Exam is worth 25% of your overall grade. Good luck and have fun!

I expect your solutions to be *well-written*, *neat*, *and organized*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

Feel free to type up your final version. The LᴬTEX source file of this exam is also available if you are interested in typing up your solutions using LᴬTEX. I'll gladly help you do this if you'd like.

The simple rules for the exam are:

1. You may freely use any theorems or problems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem X or Problem Y, then you should say so.

2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.

3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.

4. You are **NOT** allowed to copy someone else's work.

5. You are **NOT** allowed to let someone else copy your work.

6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.**

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

All but the first problem on this exam require content from Chapter 10. In particular, you are responsible for the content in 10.1, 10.3, 10.4, 10.6, 10.7, 10.8, 10.11, 10.13, 10.18, and 10.19. If you are asked to prove a theorem from Chapter 10, you are allowed to utilize any earlier result in Chapter 10 from the list in the previous sentence even if you have not proven the earlier result. For example, you may use Theorem 10.8 to prove Theorem 10.11.

1. (4 points) The *real projective plane*, denoted $\mathbb{R}P^2$, is defined to be the set of lines through the origin in $\mathbb{R}^3$. We can turn $\mathbb{R}P^2$ into a topological space by taking as a basis for the topology the collection of open cones with the cone point at the origin. Prove **one** of the following.

    (a) Show that $\mathbb{R}P^2$ is homeomorphic to the unit sphere centered at the origin in $\mathbb{R}^3$ with antipodal points identified.

    (b) Show that $\mathbb{R}P^2$ is homeomorphic to a disk with two edges on its boundary identified as in Figure 12.7 on page 173.

   Consider the unit sphere centered at the origin in $\mathbb{R}^3$. Note that this is the "shell" of unit ball.

2. (4 points each) Prove **two** of Theorem 10.6, 10.8, 10.13, 10.18.

3. (4 points) Prove Theorem 10.19.