

Homework 9

Combinatorics

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work. In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments with no questions asked**.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, Discord, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's [Academic Integrity Policy](#).

Complete the following problems.

1. Prove that a permutation $w \in S_n$ is uniquely determined by its inversion set.
2. If possible, provide an example each of the following. If no such example exists, explain why.
 - (a) A lattice L that is not a ranked poset.
 - (b) A ranked poset P that is not a lattice.
 - (c) A lattice L together with a subposet Q of L such that Q is not a lattice.
 - (d) A ranked poset P together with a subposet Q of P such that Q is not ranked.
3. Suppose (P, \leq_P) and (Q, \leq_Q) are finite posets.
 - (a) Define $\leq_{P \times Q}$ on $P \times Q$ via

$$(x, y) \leq_{P \times Q} (x', y') \text{ if and only if } x \leq_P x' \text{ and } y \leq_Q y'.$$

Prove that $(P \times Q, \leq_{P \times Q})$ is a poset.

- (b) Prove that if P and Q are ranked posets with rank function rk_P and rk_Q , respectively, then $P \times Q$ is a ranked poset with rank function

$$\text{rk}_{P \times Q}(x, y) := \text{rk}_P(x) + \text{rk}_Q(y).$$

4. Let P be a labeled poset consisting of the disjoint union of the chains $1 <_P 2 <_P \cdots <_P k$ and $k + 1 <_P k + 2 <_P \cdots <_P n$ for some k . Characterize the set of linear extensions of P . *Hint:* For $w \in \mathcal{L}(P)$, consider $\text{Des}(w^{-1})$.