Homework 10

Combinatorics of Genome Rearrangements

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work. In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments with no questions asked**.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete the following problems.

1. How many signed permutations in $S_n^{\pm}$ have exactly two breakpoints? Justify your answer.

2. Let $\pi = -6\ 1\ 2\ 3\ 4\ 5 \in S_6^{\pm}$.

   (a) How many steps will the greedy algorithm we discussed in class implement when sorting π? Justify your answer.

   (b) Find the reversal distance of π. Justify your answer.

   (c) Draw the breakpoint diagram for π. How many cycles does $\mathrm{BG}(\pi)$ have? How many components does $\mathrm{BG}(\pi)$ have? Determine which components are oriented versus unoriented.

3. Recall that if we identify the human X chromosome with the identity in S_{12}, then the mouse X chromosome is modeled using $1\ 7\ -9\ 11\ 10\ 3\ -2\ -6\ 5\ -4\ -8\ 12$. For this problem, let $\pi = 1\ 7\ -9\ 11\ 10\ 3\ -2\ -6\ 5\ -4\ -8\ 12$.

   (a) Find a sequence of 7 reversals that sorts π into the identity.

   (b) Using part (a) and our lower bound involving breakpoints, what can you conclude about $d_r(\pi)$?

   (c) Draw the breakpoint diagram for π. How many cycles does $\mathrm{BG}(\pi)$ have? How many components does $\mathrm{BG}(\pi)$ have? Determine which components are oriented versus unoriented.

4. Let $\pi = n\ (n-1)\ \cdots\ 321 \in S_n^{\pm}$.

   (a) How many steps will the greedy algorithm implement when sorting π? Justify your answer.

   (b) Explain why no single reversal applied to π will reduce the number of breakpoints.

   (c) In light of part (b) and the lower bound theorem involving breakpoints, what can you conclude about $d_r(\pi)$?

   (d) Suppose n is odd. Find an algorithm that sorts π in n reversals. In light of part (c), what can you conclude about $d_r(\pi)$ when n is odd?

(e) Suppose n is even. Find an algorithm that sorts π in $n+1$ reversals. In light of part (c), what can you conclude about $d_r(\pi)$ when n is even?

5. Let $\pi \in S_n^{\pm}$ and for $i \leq j$, let ρ_{ij} be the reversal that acts on π by reversing $\pi_i \pi_{i+1} \cdots \pi_j$ in π (and changing the signs). Prove that if both (π_{i-1}, π_i) and (π_j, π_{j+1}) are breakpoints (in π^L) but in different cycles in $\mathrm{BG}(\pi)$, then $\mathrm{cyc}(\mathrm{BG}(\pi \circ \rho_{ij})) = \mathrm{cyc}(\mathrm{BG}(\pi)) - 1$ and $\mathrm{bp}(\pi \circ \rho_{ij}) = \mathrm{bp}(\pi)$.