Homework 7

Combinatorics of Genome Rearrangements

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work. In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments with no questions asked**.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete the following problems.

1. Wrap up the proof that if $\pi \in S_n$, then
$$d_{bi}(\pi) = \frac{n+1-\mathrm{cyc}(\mathrm{DBG}(\pi))}{2}.$$

2. Let $\pi = 4162573 \in S_7$. This is the same permutation we used as an example in class.
   (a) Find $d_{bi}(\pi)$.
   (b) Find an optimal sequence of block interchanges that sorts π to the identity.

3. Let $\pi = 654321 \in S_6$.
   (a) Draw the directed breakpoint graph for π.
   (b) Find $d_{bi}(\pi)$.
   (c) Find an optimal sequence of block interchanges that sorts π to the identity.

4. Prove that
$$\mathrm{cyc}(\mathrm{DBG}(n \cdots 321)) = \begin{cases} 2, & \text{if } n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$
and then use this to prove that $d_{bi}(n \cdots 321) = \left\lfloor \frac{n}{2} \right\rfloor$.

5. Prove that
$$\max\{d_{bi}(\pi) \mid \pi \in S_n\} = \left\lfloor \frac{n}{2} \right\rfloor.$$
That is, the diameter of the sorting graph for S_n using block interchanges is $\left\lfloor \frac{n}{2} \right\rfloor$.

6. It turns out that there are 8 permutations in S_4 that attain the maximal block interchange distance. Find all 8 permutations.