

Quiz 6

Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 8 points for a total of 32 points. Good luck and have fun!

Part A

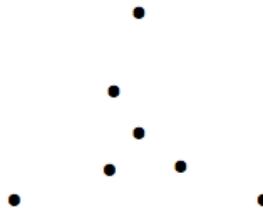
Complete **two** of the following problems.

- A1. In the senate of the Klingon home world no senator has more than three enemies. Show that the senate can be separated into two houses so that nobody has more than one enemy in the same house.
- A2. There are $2n$ Federation ambassadors invited to a Ferengi banquet. Every ambassador has at most $n - 1$ enemies. Show that the ambassadors can be seated around a round table avoiding enemies sitting next to each other.
- A3. Show that in any set of seven different positive integers there are three numbers such that the greatest common divisor of any two of them leaves the same remainder when divided by three.

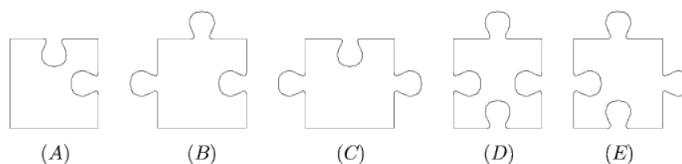
Part B

Complete **two** of the following problems.

- B1. Consider the configuration of dots given below. Two players take turns connecting two dots with a straight line segment so that the line segments do not cross each other. The player unable to move loses the game. Who has a winning strategy?



- B2. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type E are there in the puzzle? If it's not possible, explain why. *Hint: 253 is divisible by 11.*



- B3. There are n houses and n wells in a town. Show that every house can be connected to a different well so that no two connecting roads cross each other. *Note:* You don't have any control over where the houses and wells are located.