Homework 8

Discrete Mathematics

Please review the *Rules of the Game* from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions in our Q&A Discussion Board in Canvas. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to three late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. **Unless explicitly stated otherwise, you are expected to justify your answers.** In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. Prove that for all $n \in \mathbb{N}$, we have

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}.$$

2. Suppose n lines are drawn in the plane so that no two lines are parallel and no three lines intersect at any one point. Such a collection of lines is said to be in **general position**. Every collection of lines in general position divides the plane into disjoint regions, some of which are polygons with finite area (bounded regions) and some of which are not (unbounded regions). For Part (c), consider using the results from Parts (a) and (b).

   (a) Let $R(n)$ be the number of regions the plane is divided into by n lines in general position. Conjecture a formula for $R(n)$ and prove that your conjecture is correct.

   (b) Let $U(n)$ be the number of unbounded regions the plane is divided into by n lines in general position. Conjecture a formula for $U(n)$ and prove that your conjecture is correct.

   (c) Let $B(n)$ be the number of bounded regions the plane is divided into by n lines in general position. Conjecture a formula for $B(n)$ and prove that your conjecture is correct.

   (d) Suppose we color each of the regions (bounded and unbounded) so that no two adjacent regions (i.e., share a common edge) have the same color. What is the fewest colors we could use to accomplish this? Prove your assertion.

3. Whoziwhatzits come in boxes of 6, 9, and 20. Prove that for any natural number $n \geq 44$, it is possible to buy exactly n Whoziwhatzits with a combination of these boxes.

4. Prove that the number of binary strings of length n that never have two consecutive 1's is the Fibonacci number f_{n+2}. See Problem 5.11 from our textbook for the definition of the Fibonacci numbers.