Homework 13

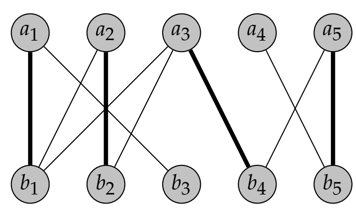
Discrete Mathematics

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please box your final answer.

1. As we have seen, some bipartite graphs have a total matching and some do not. We say that a bipartite graph $G = (V, E)$ has a **partial matching** if there is subset $F \subseteq E$ of edges such that a vertex is the endpoint of at most one edge of F. Certainly, every total matching is a partial matching, but a partial matching may not be a total matching. Every bipartite graph with at least one edge has a partial matching, so we can look for the largest partial matching in a graph (i.e., a partial matching involving the largest number of edges).

   Bob claims that he has found the largest partial matching for the graph below (his matching is in bold). He explains that no other edge can be added, because all the edges not used in his partial matching are connected to matched vertices. Is he correct? Explain.



2. Consider the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ given by the following data.

   G_1: $V_1 = \{a, b, c, d, e, f, g\}$
   $E_1 = \{\{a,b\},\{a,d\},\{b,c\},\{b,d\},\{b,e\},\{b,f\},\{c,g\},\{d,e\},\{e,f\},\{f,g\}\}$

   G_2: $V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$
   $E_2 = \{\{v_1,v_4\},\{v_1,v_5\},\{v_1,v_7\},\{v_2,v_3\},\{v_2,v_6\},\{v_3,v_5\},\{v_3,v_7\},\{v_4,v_5\},\{v_5,v_6\},\{v_5,v_7\}\}$

   (a) Let $m : V_1 \to V_2$ be a function given by

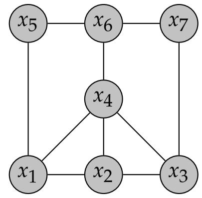
   $$m(a) = v_4, m(b) = v_5, m(c) = v_1, m(d) = v_6, m(e) = v_2, m(f) = v_3, m(g) = v_7.$$

   Is m an isomorphism between G_1 and G_2? Explain.

   (b) Using the ordering $v_1, v_2, v_3, v_4, v_5, v_6, v_7$, write down the adjacency matrix for G_2.

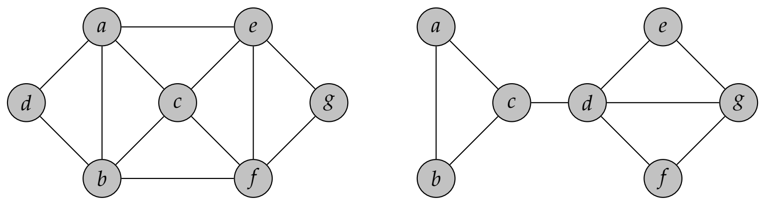
   (c) Find an ordering on the vertices of V_1 and write down the corresponding adjacency matrix for G_1 that makes it clear that $G_1 \cong G_2$.

   (d) Is the graph G_3 given below isomorphic to G_1 and G_2? Explain.



G_3

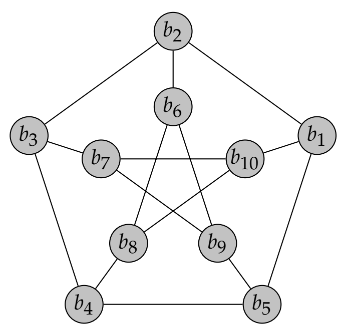
3. Determine whether each of the following graphs has an Euler circuit, an Euler trail that is not a circuit, or neither. If the graph has an Euler circuit or an Euler trail that is not a circuit, find one. Otherwise, explain why neither exists.



G H

4. For which m and n does the graph $K_{m,n}$ contain an Euler circuit? An Euler trail that is not a circuit?

5. Determine whether the following graph has a Hamilton path that is not a cycle and determine whether is has a Hamilton cycle. Explain.



6. If possible, find an example of a graph that has a Hamilton cycle but not an Euler circuit. If no such example exists, explain why.

7. If possible, find an example of a graph that has an Euler circuit but not a Hamilton cycle. If no such example exists, explain why.

8. Find 3 spanning trees of the wheel graph W_6 such that no pair of the spanning trees is isomorphic.