Homework 7

Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition. You are highly encouraged to seek out assistance by asking questions on Discord. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please ⎡box⎤ your final answer.

1. A **weak composition** of n into k parts is an ordered list of k nonnegative integers that sum to n. Notice that we allow a part to be 0. How many weak compositions of n into k parts are there? Justify your answer.

2. What is the coefficient of $x^3 y^5$ in the expansion of $(2x - 3y)^8$? No justification required.

3. Use the Binomial Theorem to determine what the sum $\sum_{k=0}^{n} (-2)^k \binom{n}{k}$ equal to.

4. Find the alternating row sums in Pascal's Triangle. That is, for $n \geq 0$, find a formula for

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} := \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n}.$$

   Hint: The answer for $n = 0$ is different than for $n > 0$. For $n > 0$, you can either use the Binomial Theorem or make use of Pascal's Identity together with careful algebraic bookkeeping.

5. At the end of the semester a professor must award all students a grade of A, B, C, D, or F. How big must the class be to guarantee that at least 5 students will earn the same grade?

6. A cash box contains eight \$1 bills, six \$5 bills, five \$10 bills, and three \$20 bills. If I pull bills out at random:

(a) How many bills must I pull to be guaranteed at least 4 of one denomination?

(b) How many bills must I pull to be guaranteed at least 5 of one denomination?

7. Use the Pigeonhole Principle to explain why if five points are placed inside an equilateral triangle of side length 1, then at least two of them are within 1/2 unit of each other.