Homework 8

Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition. You are highly encouraged to seek out assistance by asking questions on Discord. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. In any group of 6 people, prove that there is either a group of 3 who are known to each other or there are three who are total strangers. *Hint:* Pick one person, say person A, and think about the 5 others. Then by Pigeonhole Principle either at least 3 are known to A or at least 3 are strangers to A. Pick one of these scenarios (as the argument for the other is similar) and argue from there.

2. Of 200 aliens at an interplanetary convention, 83 had pointed ears, 101 had bony brows and 73 ridged noses. If 32 had pointed ears and ridged noses, 52 had pointed ears and bony brows, 34 had bony brows and ridged noses and 22 had all three features, how many did not have any of the three features?

3. Of 47 students in a dorm, 34 like hiking, 23 like mountain biking and 15 like rock climbing. If 12 like to hike and bike, 13 like to hike and rock climb, 7 enjoy all of these things and everyone likes at least one thing.

   (a) How many like to mountain bike and rock climb?

   (b) How many like to mountain bike but not rock climb?

4. How many nonnegative integer solutions are there for the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 32$ such that $x_1 \leq 6, x_2 \leq 7, x_4 \leq 5$, and $x_6 \leq 8$?

5. POSTPONED TO NEXT HOMEWORK ASSIGNMENT.

6. POSTPONED TO NEXT HOMEWORK ASSIGNMENT.