Name:

Names of Any Collaborators:

Instructions

This portion of Exam 1 is worth a total of 22 points and is due at the beginning of class on **Monday, February 20**. Your total combined score on the in-class portion and take-home portion is worth 15% of your overall grade. I expect your solutions to be *well-written, neat, and organized*. Do not turn in rough drafts. What you turn in should be the "polished" version of potentially several drafts.

Feel free to type up your final version. The LaTeX source file of this exam is also available if you are interested in typing up your solutions using LaTeX. I'll gladly help you do this if you'd like.

The simple rules for the exam are:

1. You may freely use any results that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem xyz, then you should say so.
2. Unless you prove them, you cannot use any results that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.
4. You are **NOT** allowed to copy someone else's work.
5. You are **NOT** allowed to let someone else copy your work.
6. You are allowed to discuss the problems with each other and critique each other's work.

I will vigorously pursue anyone suspected of breaking these rules.

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.**

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

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1. (3 points each) On the in-class portion of Exam 1, you selected five of the following, where your task was to determine whether each statement is *True* or *False* and to justify your answer. Now, complete the remaining **two** that you chose not to complete during the in-class portion of the exam. If a statement is true, write a short proof. If a statement is false, provide a specific counterexample or potentially a detailed explanation why the statement cannot be true. Make sure you clearly indicate whether you believe the statement is *True* or *False*.

   (a) The sum of any four consecutive integers is divisible by four.

   (b) There exists $n \in \mathbb{N}$ such that $n^2 - 4 = 0$.

   (c) For all $n \in \mathbb{Z}$, there exists $m \in \mathbb{Z}$ such that $n + m = 0$.

   (d) There exists integers a and b such that $2a + 7b = 1$.

   (e) Let $a, b, c \in \mathbb{Z}$. If a divides bc, then either a divides b or a divides c.

   (f) Let $x, y, z \in \mathbb{N}$. If $x + y$ is odd and $y + z$ is odd, then $x + z$ is odd.

   (g) There do not exist integers m and n such that $4m - 6n = 13$.

2. (4 points each) On the in-class portion of Exam 1, you selected one of the following theorems to prove. Now, prove the remaining **two** theorems that you chose not to prove during the the in-class portion of the exam.

   Theorem B.1. For every $z \in \mathbb{Z}$, if z is odd, then 8 divides $z^2 - 1$.

   Theorem B.2. Let $a, b, c, x, y \in \mathbb{Z}$. If a divides b and a divides c, then a divides $bx + cy$.

   Theorem B.3. Let $a, b \in \mathbb{Z}$ and let $n \in \mathbb{N}$. If a divides b, then a^n divides b^n.

3. (4 points each) Prove **two** of the following theorems. You need to make it clear which proofs you want me to grade. Otherwise, I will grade the first one that I see.

   Theorem C.1. If x, y, and z are integers and $x + y$ and $y + z$ are both even, then $x + z$ is even.

   Theorem C.2. If there exists an odd natural number $n > 5$ such that n is not the sum of three prime numbers, then there exists an even natural number $m > 2$ such that m is not the sum of two prime numbers.*

   Theorem C.3. If x and y are positive real numbers, then $\dfrac{x+y}{2} \geq \sqrt{xy}$.

   Theorem C.4. For all $x, y \in \mathbb{R}$, $|x + y| \leq |x| + |y|$.

*Recall that a prime number is a natural number greater than or equal to 2 that has exactly two natural number divisors. This problem is related to the Goldbach Conjecture, proposed by Christian Goldbach in 1742, which says that every even number greater than 2 is the sum of two prime numbers. No one knows whether the Goldbach Conjecture is true or false, but a proof one way or the other has a million dollar prize. Fortunately, you don't have to prove Goldbach's Conjecture to do this problem.

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